

Algorithms and Data Structures

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Plan of the lecture

- Shell Sort
- Partitioning
- Quicksort
- Binary Trees
 - Tree terminology
 - Basic Binary Tree operations
 - Finding a Node
 - Inserting a Node
 - Traversing the Tree
 - Finding maximum and minimum values
 - *Deleting a Node*
 - The efficiency of Binary Trees

Shell Sort

A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈
----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------

Array step={1,2,4}; step = 4

A ₁	A ₅
----------------	----------------

A ₂	A ₆
----------------	----------------

A ₃	A ₇
----------------	----------------

A ₄	A ₈
----------------	----------------

Shell Sort

A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈
----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------

Array step={1,2,4}; step = 2

A ₁	A ₃	A ₅	A ₇
----------------	----------------	----------------	----------------

A ₂	A ₄	A ₆	A ₈
----------------	----------------	----------------	----------------

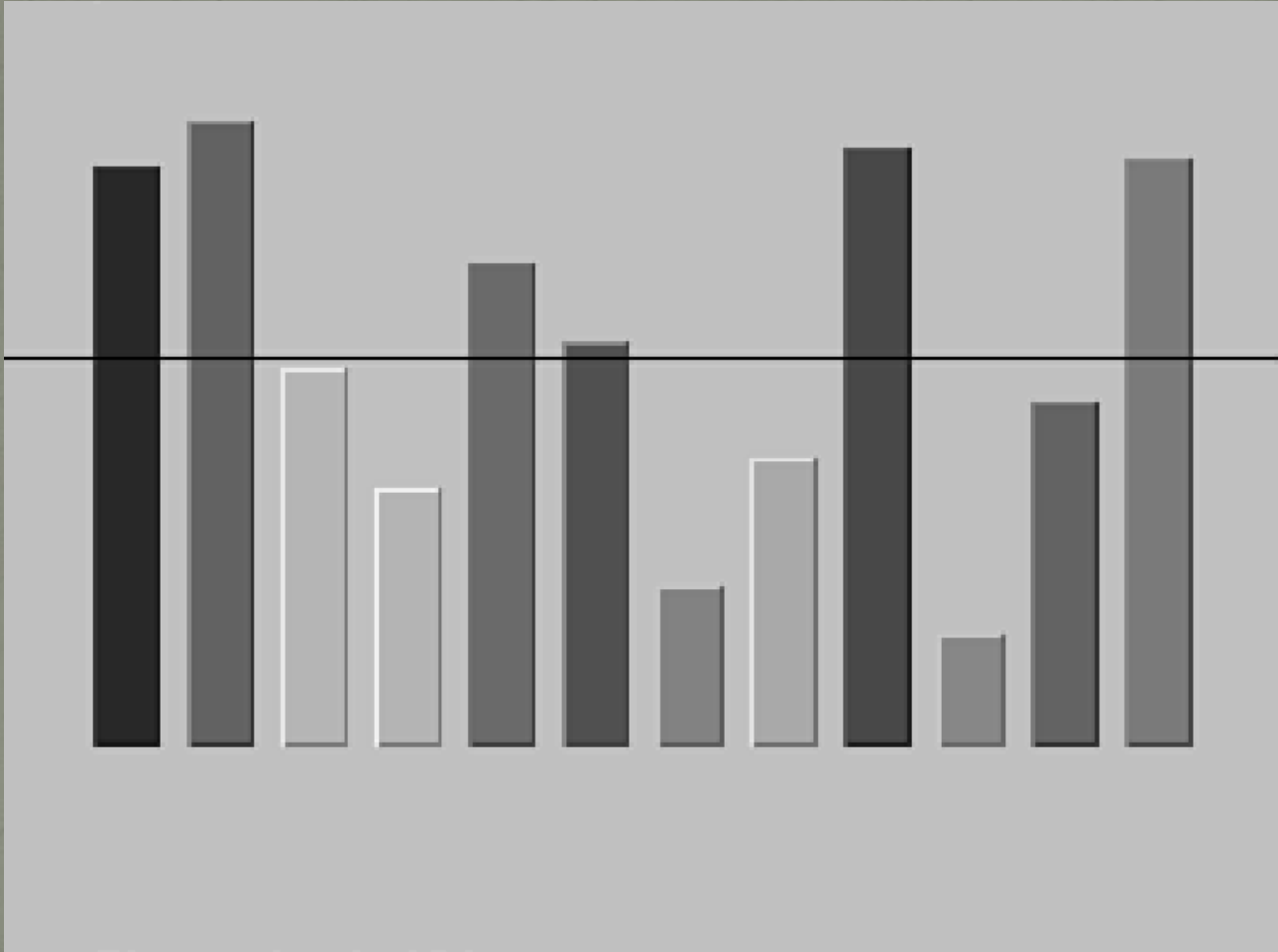
Shell Sort

A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈
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Array step={1,2,4}; step = 1

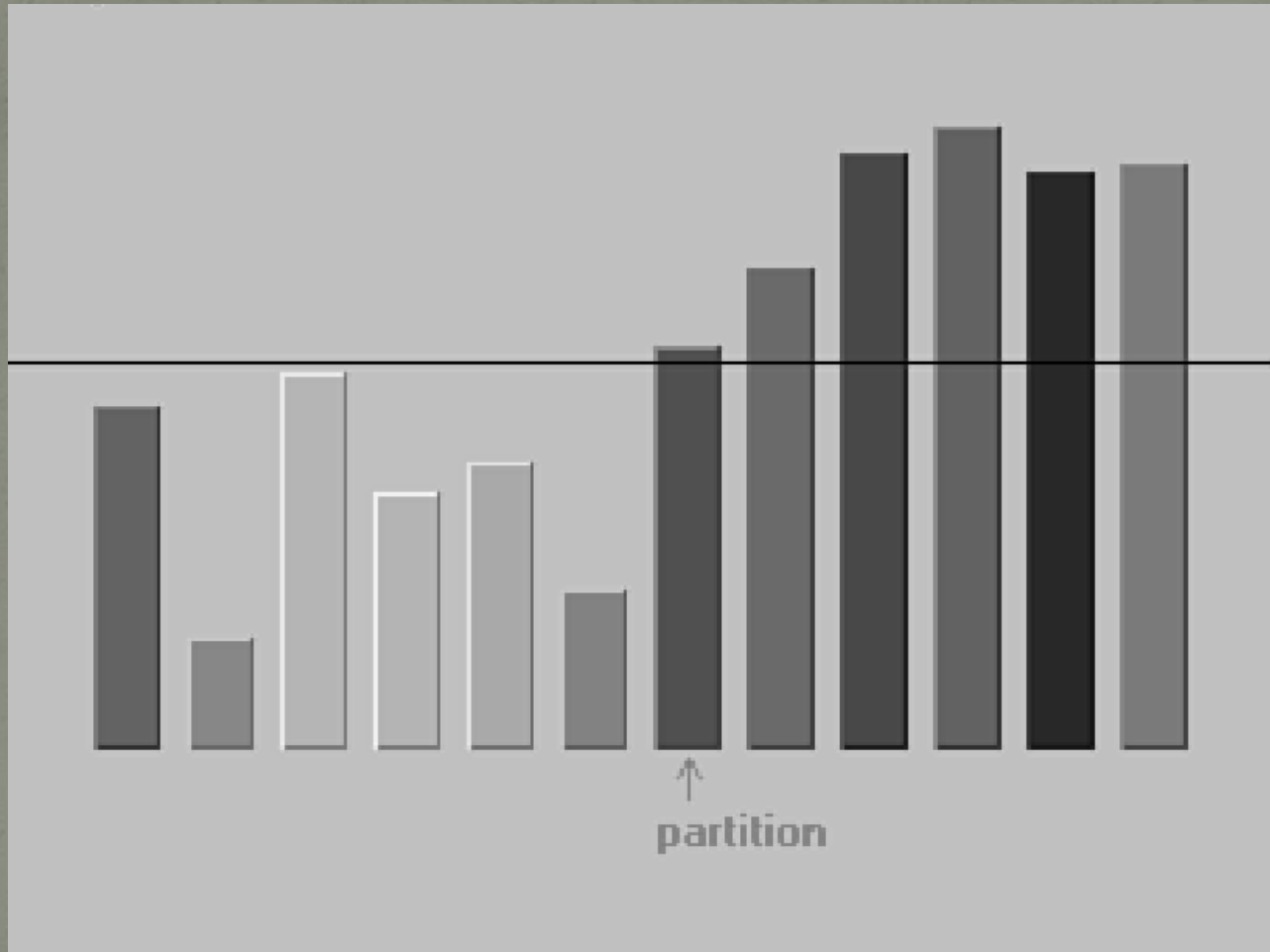
A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈
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Partitioning



(example from R. Lafore book)

Partitioning



(example from R. Lafore book)

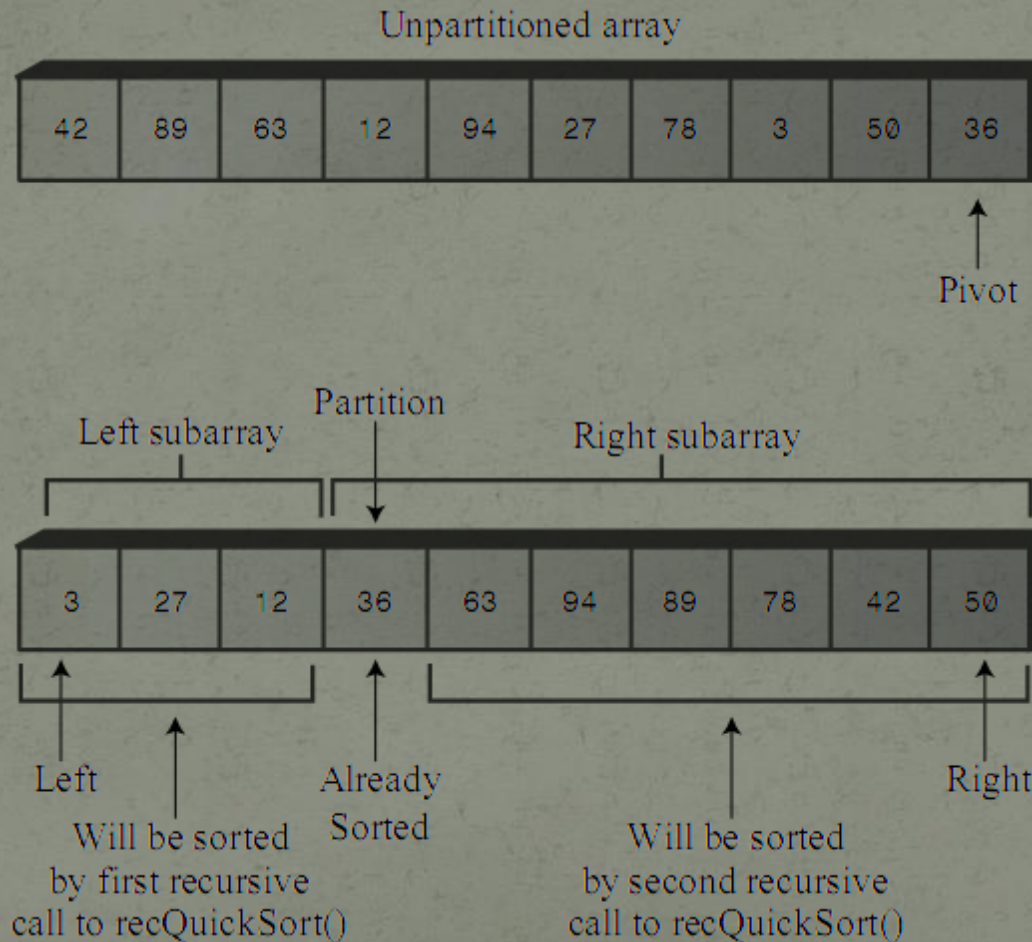
Quicksort

```
void recQuickSort(int left, int right)
{
    if(right-left <= 0)           //if size is 1,
        return;                  //    it's already sorted
    else                          //size is 2 or larger
    {
                                                //partition range
        int partition = partitionIt(left, right);

        recQuickSort(left, partition-1);    //sort left side
        recQuickSort(partition+1, right);   //sort right side
    }
}
```

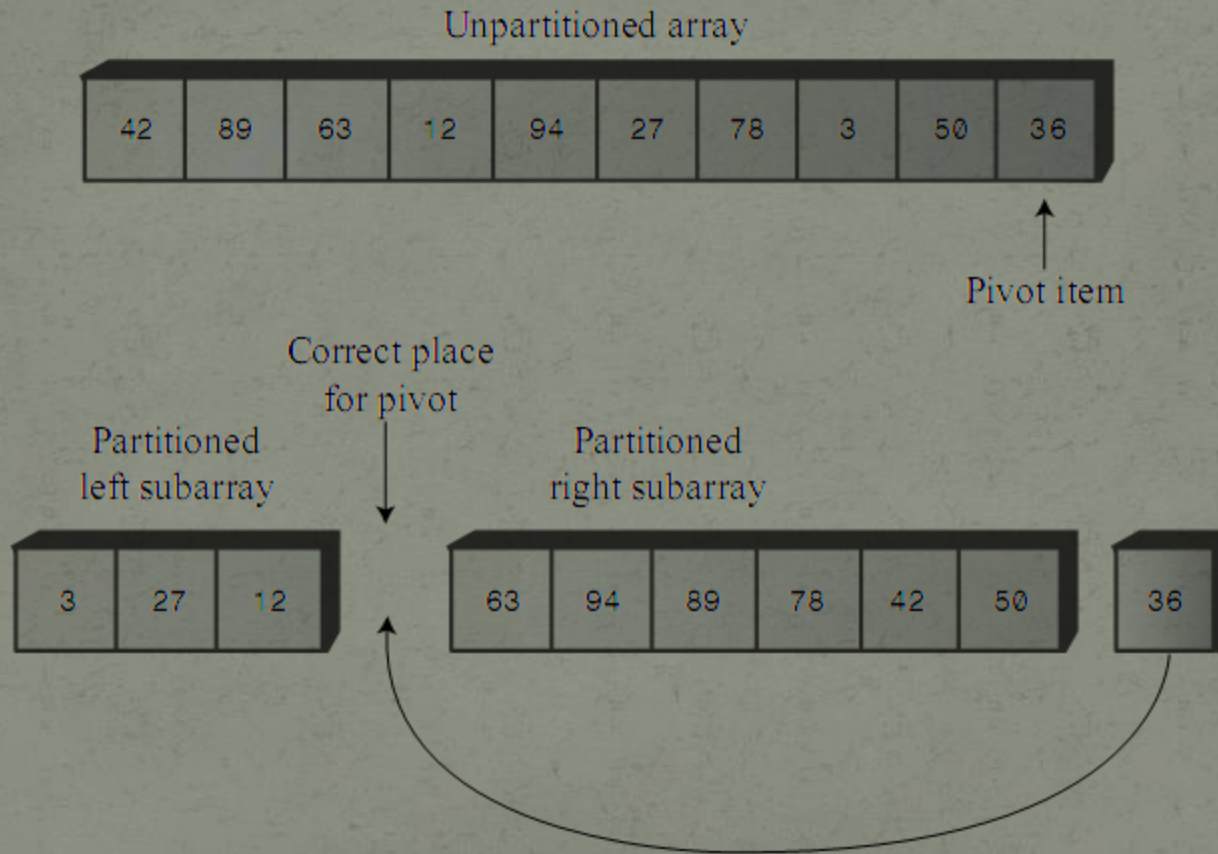
(example from R. Lafore book)

Quicksort



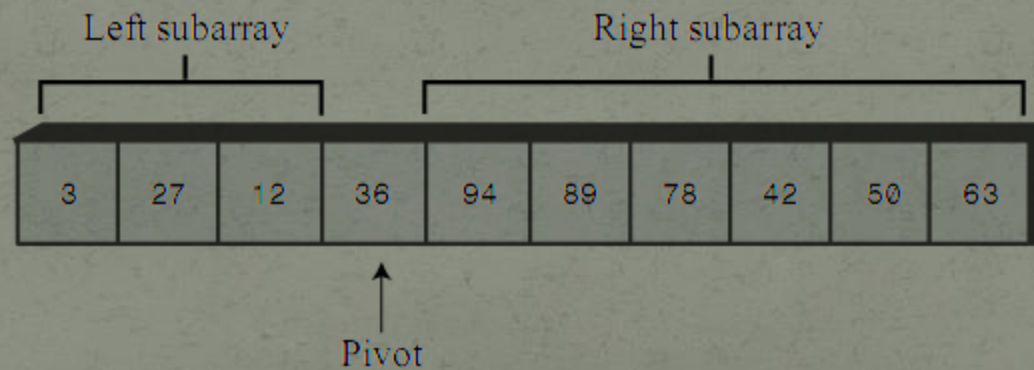
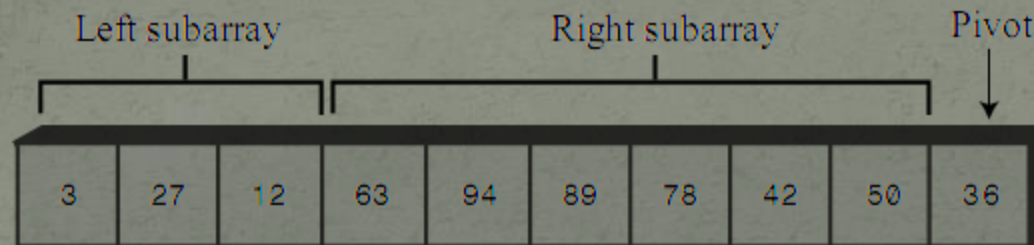
(example from R. Lafore book)

Quicksort



(example from R. Lafore book)

Quicksort



(example from R. Lafore book)

Quicksort: efficiency

- Quicksort operating in $O(N \cdot \log N)$ time (for sorting in memory) is the fastest method in majority simulations)

(example from R. Lafore book)

Binary Trees

Tree terminology

- **Path** – sequence of nodes: walking from node to node along the edges
- **Root** – the node at the top of the tree
- **Parent** – node above chosen one (except the root)
- **Child** – a node being a „child” of the parent node (connected by line)
- **Leaf** – a node without a children,
- **Subtree** – a node considerate to be a root of sub-tree.
- **Visiting** – action during visiting a node (checking the key value, displaying it, etc.); without action it is merely passing over a node.

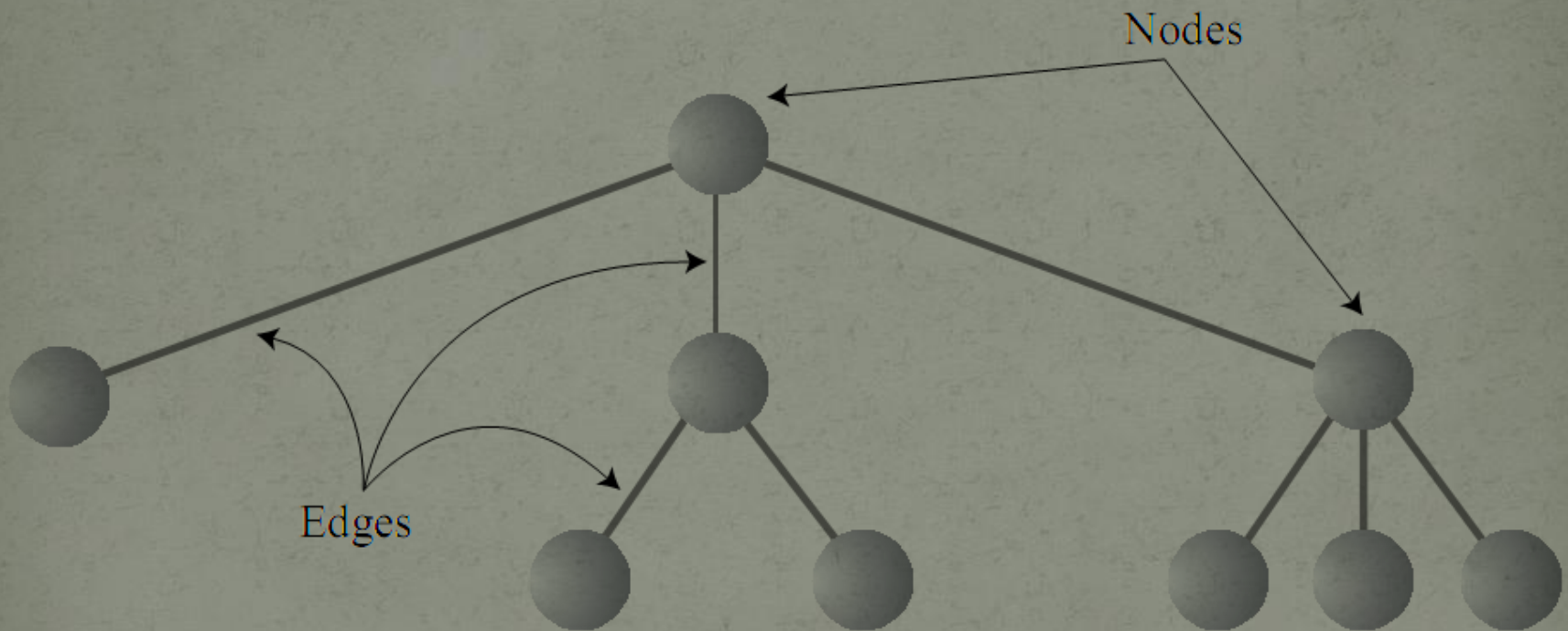
(example from R. Lafore book)

Tree terminology

- **Traversing** – to visit all nodes in some order (inorder, preorder, postorder)
- **Levels** – level refers to a number which tells us how many generations of nodes are from the root (to particular node)
- **Keys** – key value(s) stored in the node
- **Binary trees** – a node can have no more than two children

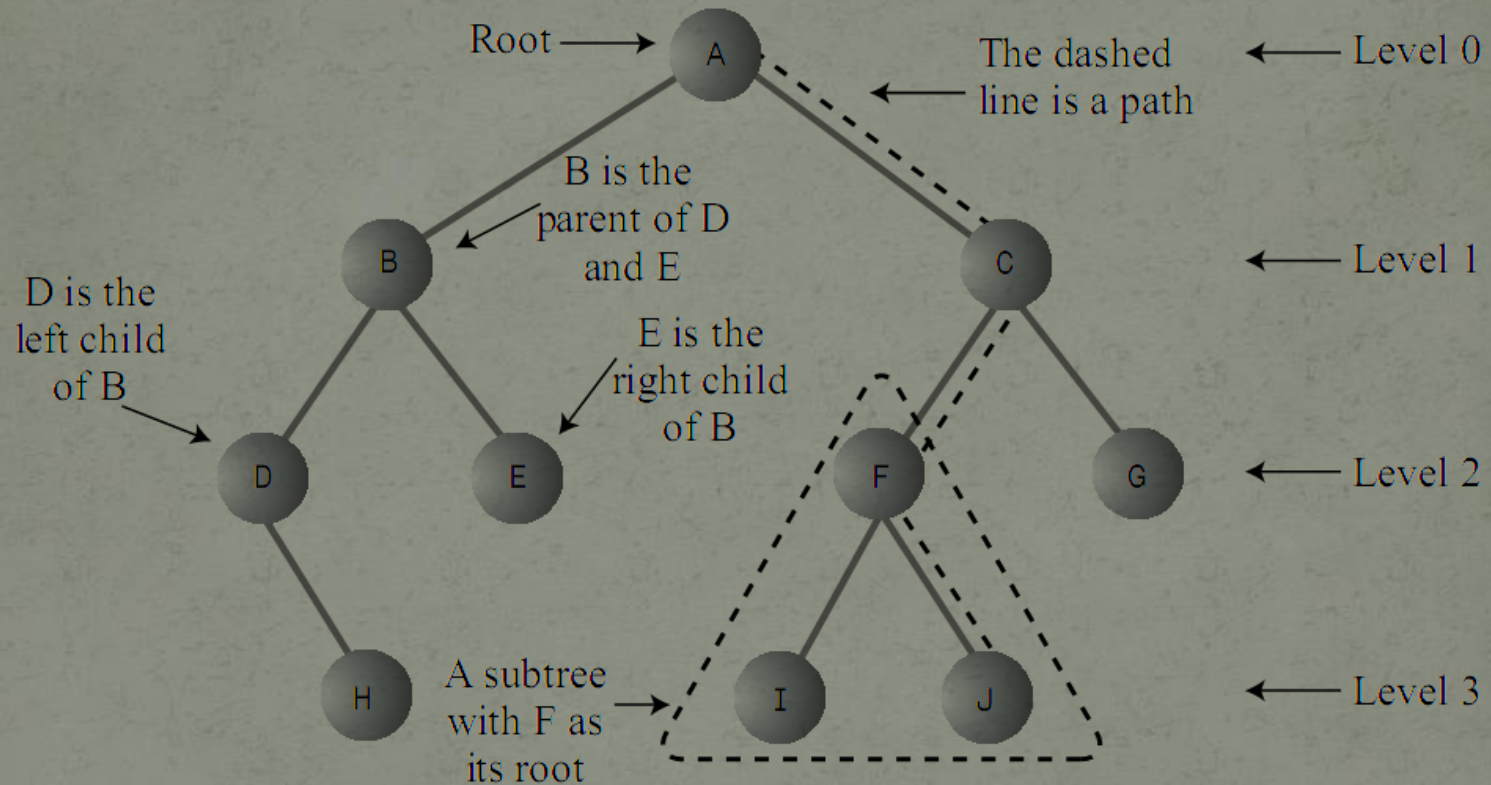
(example from R. Lafore book)

Tree terminology



(example from R. Lafore book)

Tree terminology

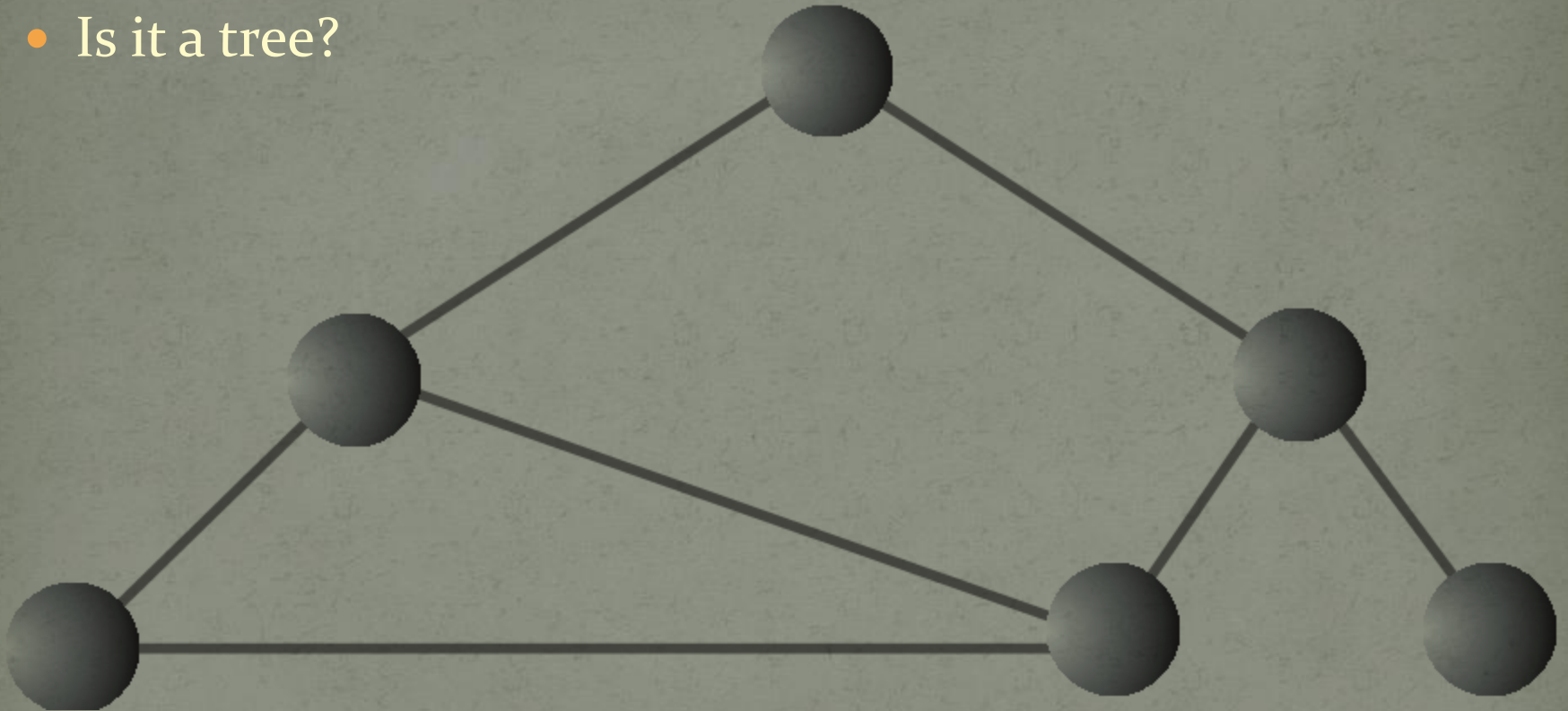


H, E, I, J, and G are leaf nodes

(example from R. Lafore book)

Tree terminology

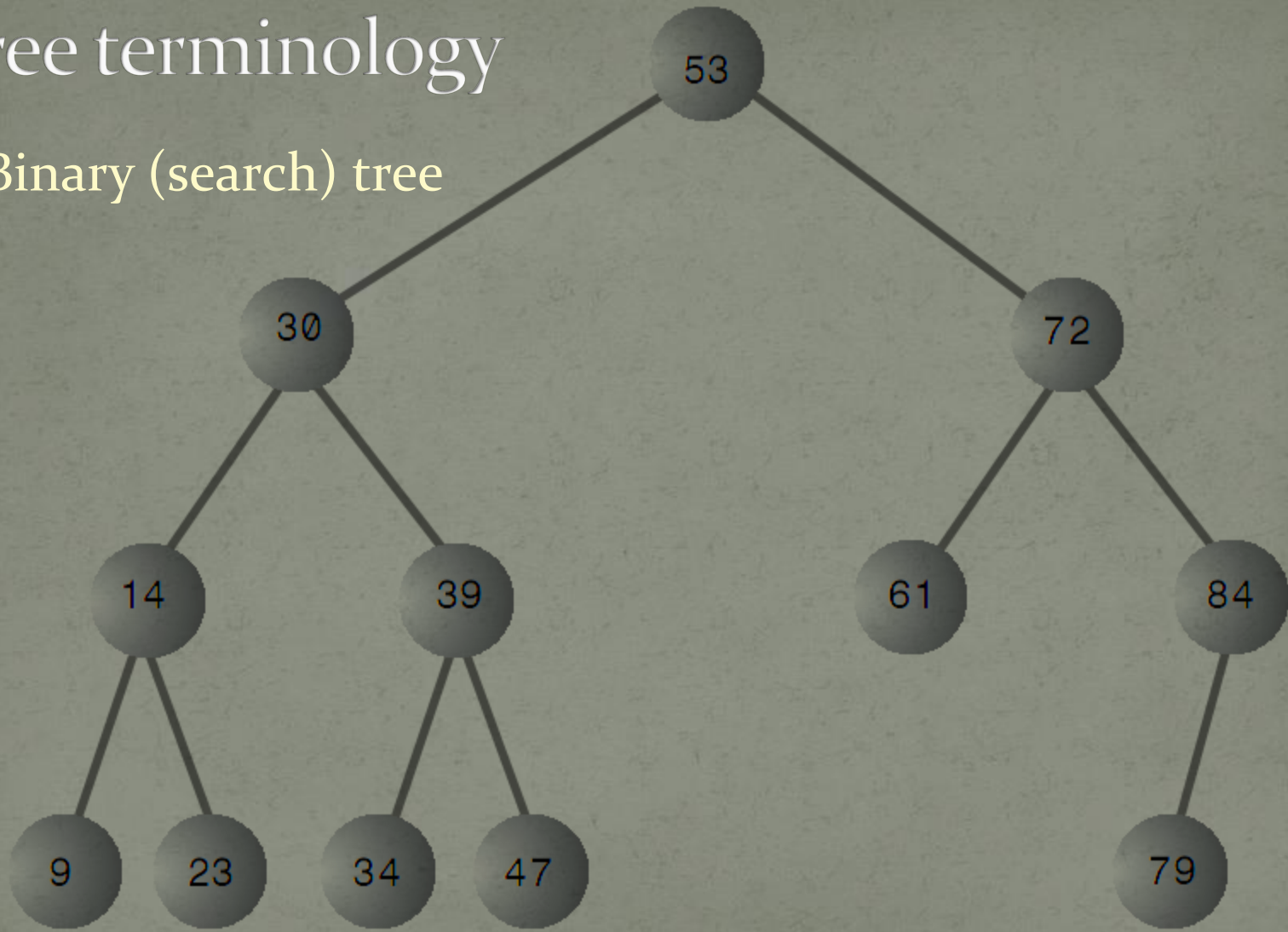
- Is it a tree?



(example from R. Lafore book)

Tree terminology

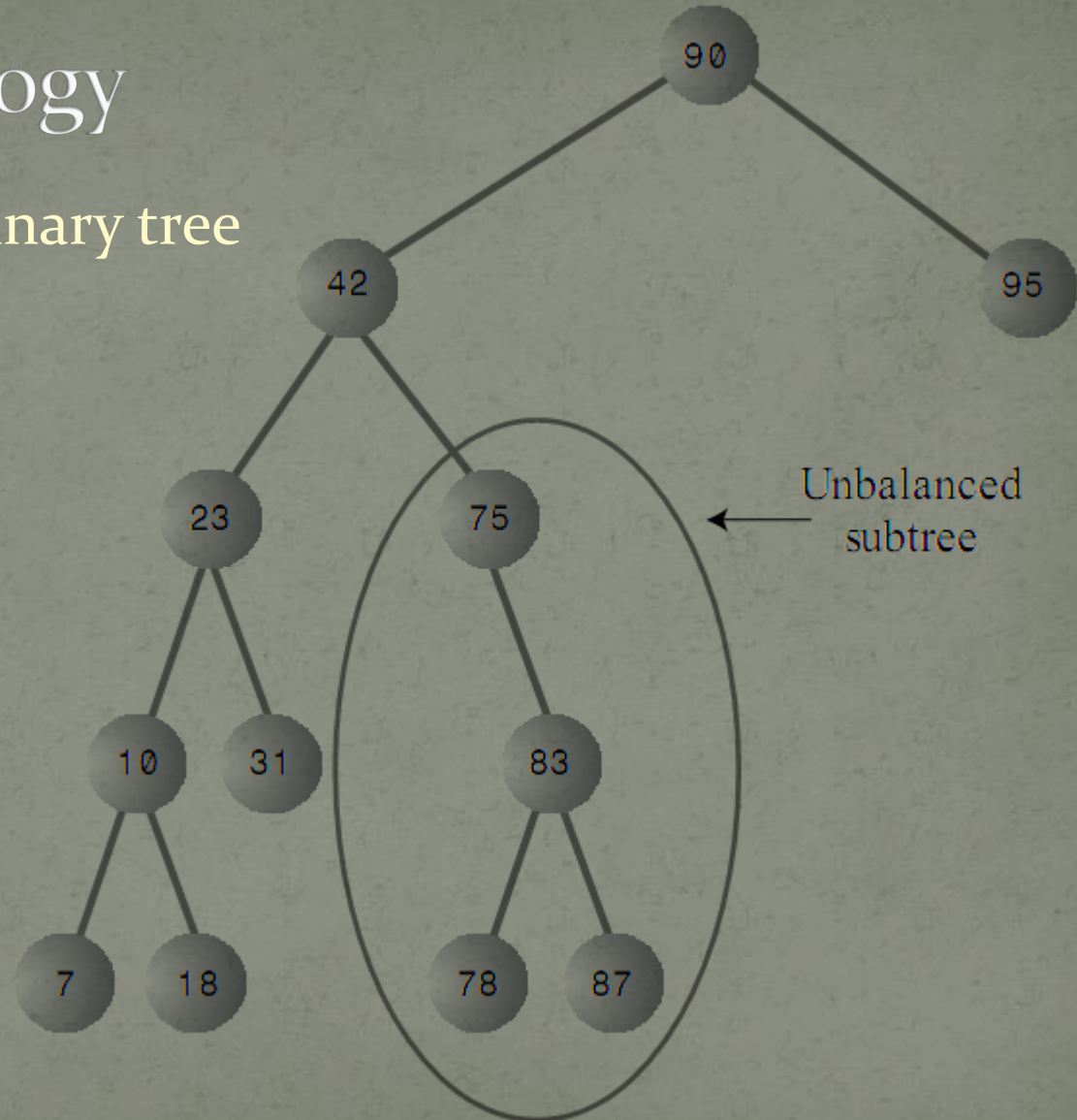
- Binary (search) tree



(example from R. Lafore book)

Tree terminology

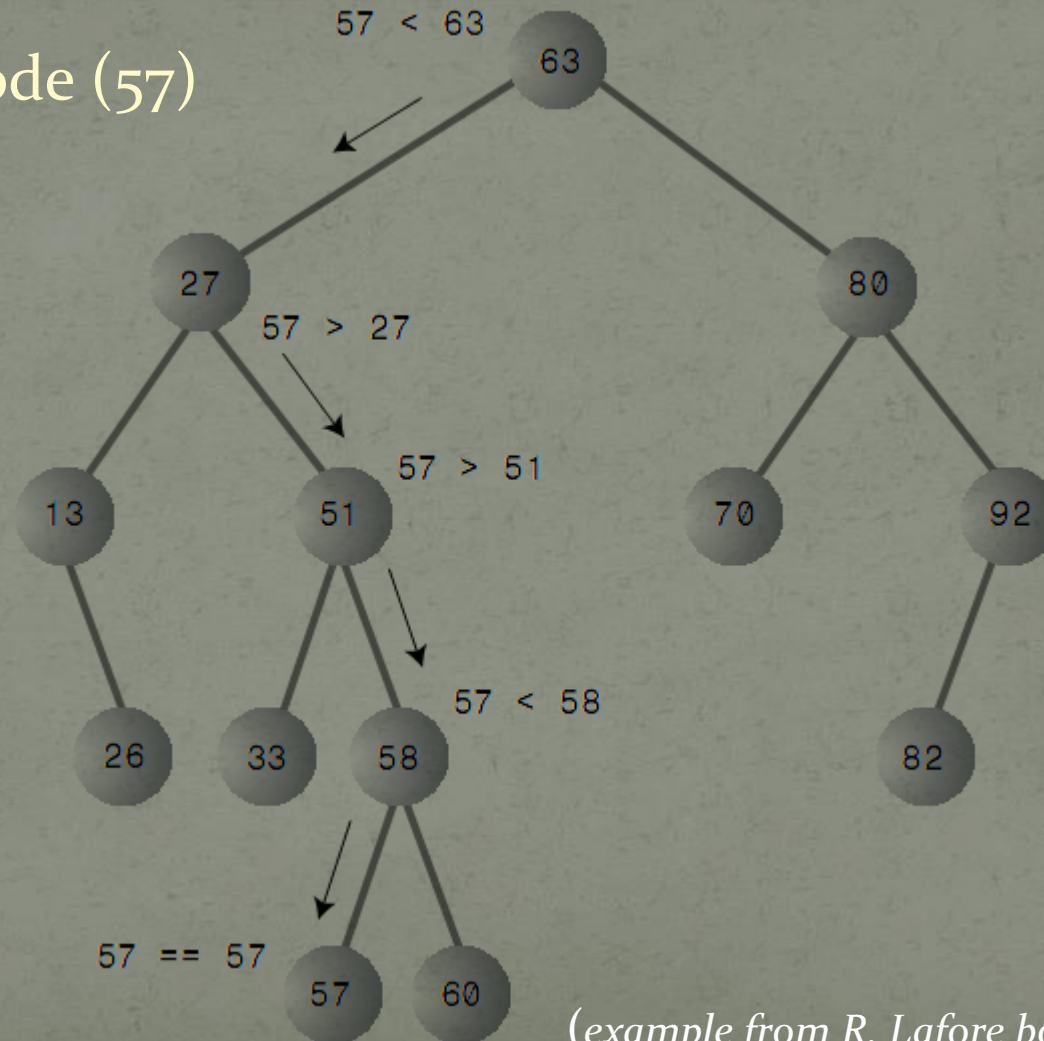
- An unbalanced binary tree



(example from R. Lafore book)

Basic Binary Tree operations

- Finding a Node (57)



(example from R. Lafore book)

Basic Binary Tree operations

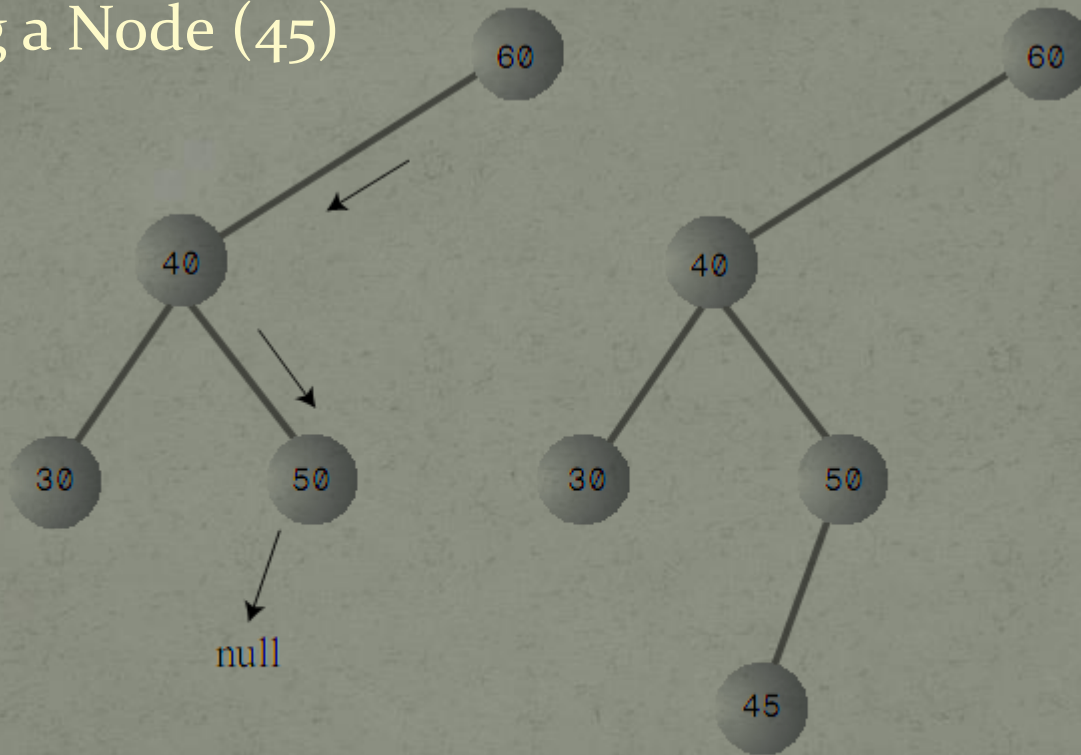
- Finding a Node (57)

```
Node* find(int key)           //find node with given key
{                             //(assumes non-empty tree)
    Node* pCurrent = pRoot;    //start at root
    while(pCurrent->iData != key) //while no match,
    {
        if(key < pCurrent->iData) //go left?
            pCurrent = pCurrent->pLeftChild;
        else //or go right?
            pCurrent = pCurrent->pRightChild;
        if(pCurrent == NULL) //if no child,
            return NULL;     //didn't find it
    }
    return pCurrent;         //found it
} //end find()
```

(example from R. Lafore book)

Basic Binary Tree operations

- Inserting a Node (45)



a) Before insertion

b) After insertion

(example from R. Lafore book)

Basic Binary Tree operations

- Inserting a Node (45)

```
void insert(int id, double dd) //insert new node
{
    Node* pNewNode = new Node;           //make new node
    pNewNode->iData = id;                  //insert data
    pNewNode->dData = dd;
    if(pRoot==NULL)                       //no node in root
        pRoot = pNewNode;
    else                                   //root occupied
    {
        Node* pCurrent = pRoot;          //start at root
        Node* pParent;
        while(true)                       //(exits internally)
        {
            pParent = pCurrent;
            if(id < pCurrent->iData)        //go left?
            {
                pCurrent = pCurrent->pLeftChild;
                if(pCurrent == NULL)        //if end of the line,
                {                           //insert on left
                    pParent->pLeftChild = pNewNode;
                    return;
                }
            } //end if go left
            else                             //or go right?
            {
```

(example from R. Lofore book)

Basic Binary Tree operations

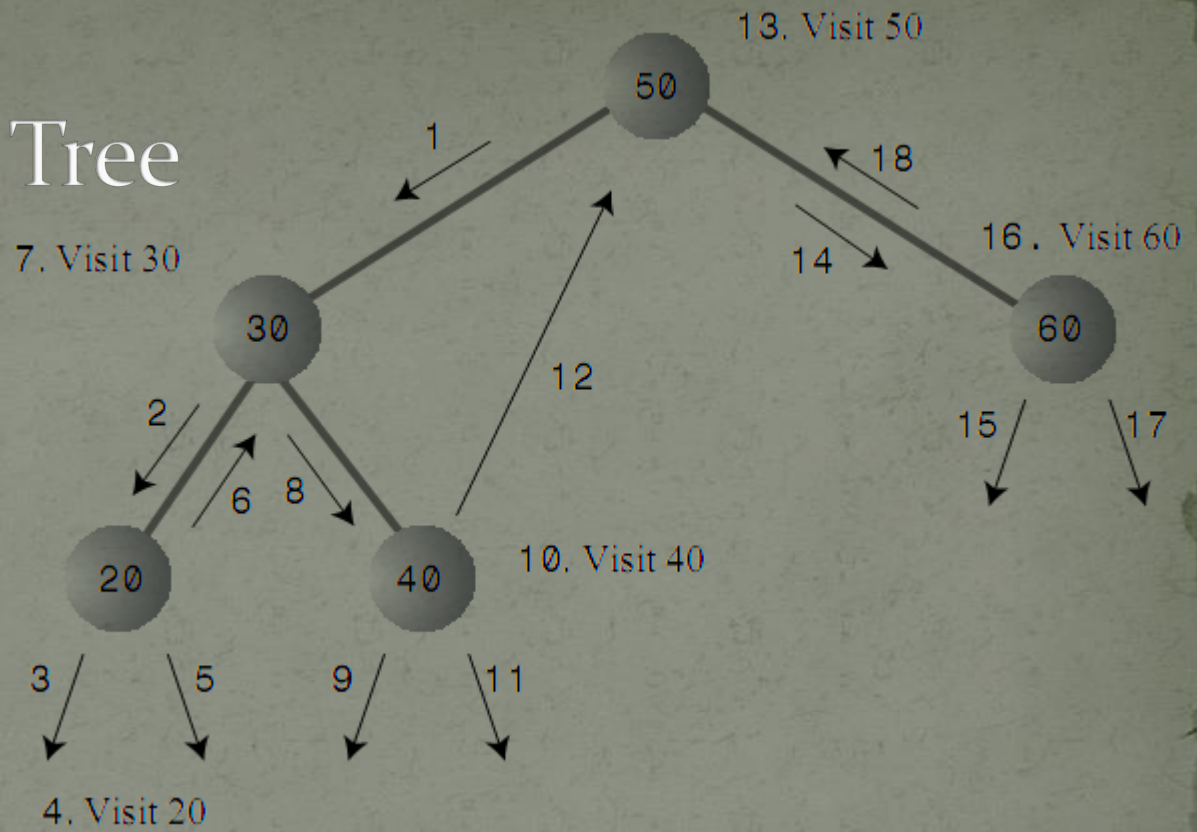
- Inserting a Node (45)

```
        else                                     //or go right?
        {
            pCurrent = pCurrent->pRightChild;
            if(pCurrent == NULL)                 //if end of the line
            {                                     //insert on right
                pParent->pRightChild = pNewNode;
                return;
            }
        } //end else go right
    } //end while
} //end else not root
} //end insert()
```

(example from R. Lafore book)

Traversing the Tree

- inOrder()
- preOrder()
- postOrder()

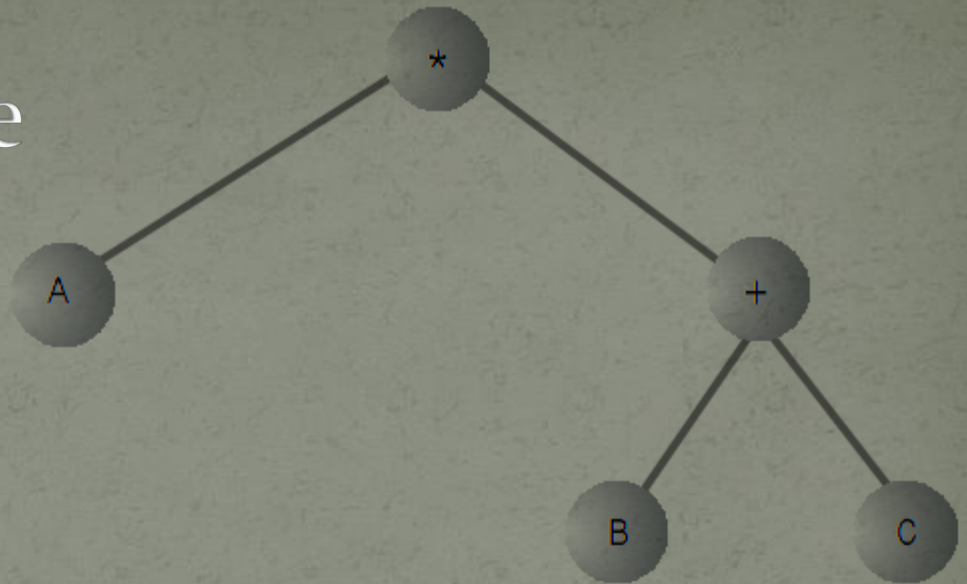


```
void inOrder(Node* pLocalRoot)
{
    if(pLocalRoot != NULL)
    {
        inOrder(pLocalRoot->pLeftChild);    //left child
        cout << pLocalRoot->iData << " ";    //display node
        inOrder(pLocalRoot->pRightChild);    //right child
    }
}
```

(example from R. Lafore book)

Traversing the Tree

- inOrder()
- preOrder()
- postOrder()



Infix: A * (B + C)

Prefix: *A + BC

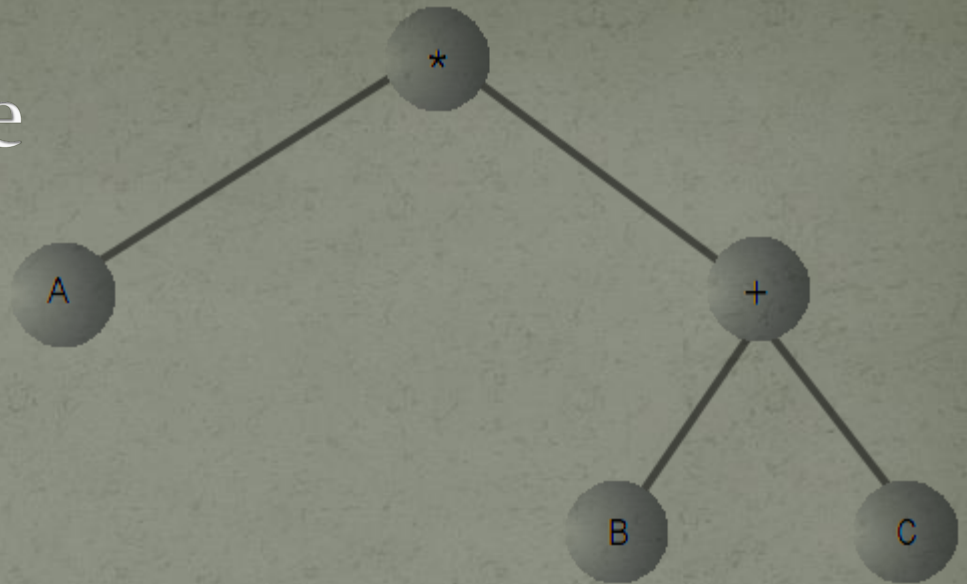
Postfix: ABC + *

```
void preOrder(Node* pLocalRoot)
{
    if(pLocalRoot != NULL)
    {
        cout << pLocalRoot->iData << " ";    //display node
        preOrder(pLocalRoot->pLeftChild);    //left child
        preOrder(pLocalRoot->pRightChild);    //right child
    }
}
```

(example from R. Lafore book)

Traversing the Tree

- inOrder()
- preOrder()
- postOrder()



Infix: A * (B + C)

Prefix: *A + BC

Postfix: ABC + *

```
void postOrder(Node* pLocalRoot)
{
    if(pLocalRoot != NULL)
    {
        postOrder(pLocalRoot->pLeftChild);    //left child
        postOrder(pLocalRoot->pRightChild);    //right child
        cout << pLocalRoot->iData << " ";    //display node
    }
}
```

(example from R. Lafore book)

Finding maximum and minimum values

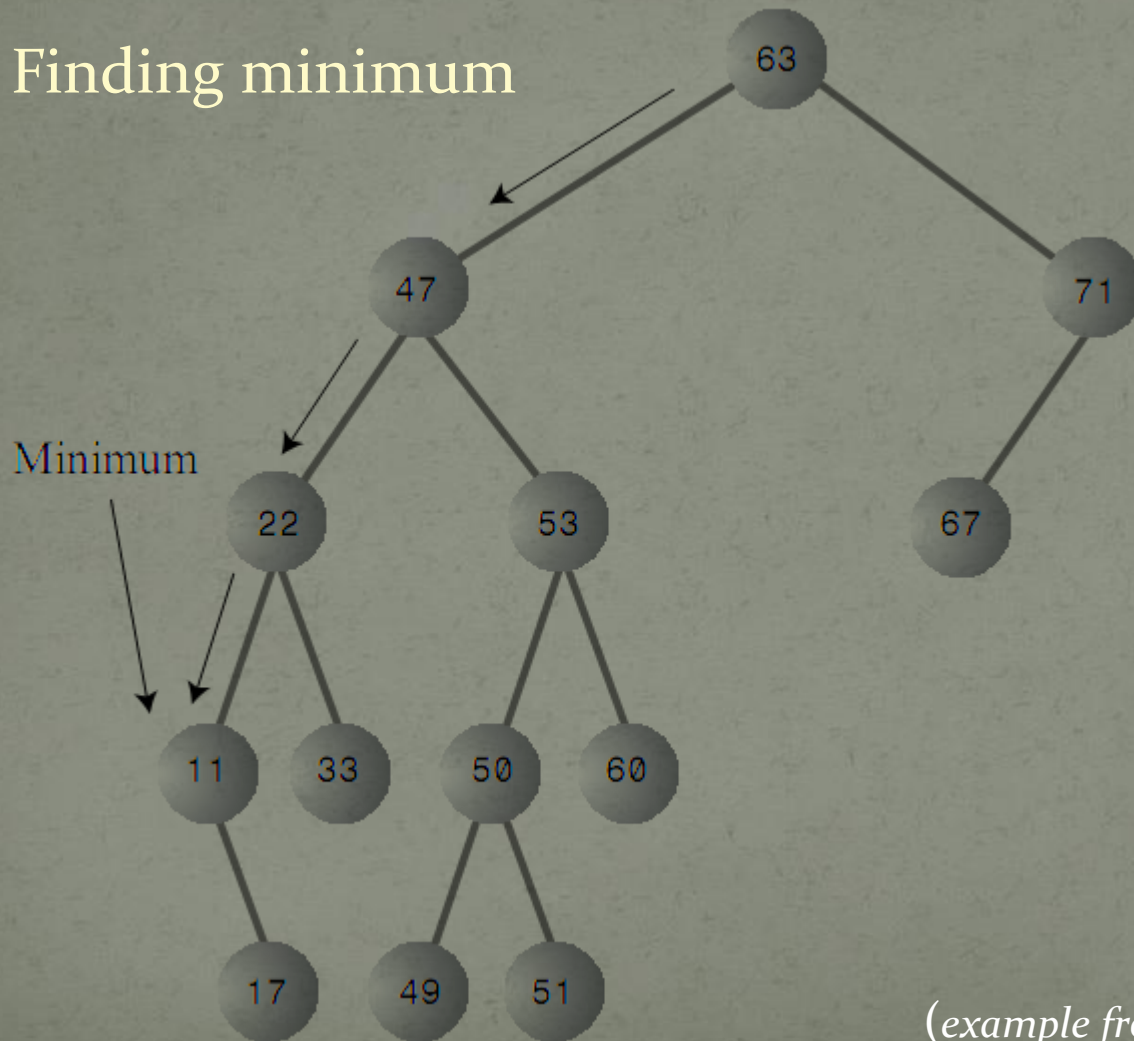
- Finding minimum

```
Node* minimum()      // returns node with minimum key value
{
    Node* pCurrent, pLast;
    pCurrent = pRoot;           //start at root
    while(pCurrent != NULL)     //until the bottom,
    {
        pLast = pCurrent;       //remember node
        pCurrent = pCurrent->pLeftChild; //go to left child
    }
    return pLast;
}
```

(example from R. Lafore book)

Finding maximum and minimum values

- Finding minimum



(example from R. Lafore book)

The efficiency of Binary Trees

- Operations like finding a particular node involve descending the tree from level 0 to level with search node. How long (many operations) it will take to do this? (for full tree)

(example from R. Lafore book)

The efficiency of Binary Trees

- During a search we need to visit a one node per level.

<i>Number of Nodes</i>	<i>Number of Levels</i>	
1	1	$N=2^L - 1 \quad // +1$
3	2	$N+1=2^L$
7	3	$L=\log_2(N+1)$
15	4	
31	5	$O(\log N)$
...	...	
1,023	10	
...	...	
32,767	15	
...	...	
1,048,575	20	

(example from R. Lafore book)

