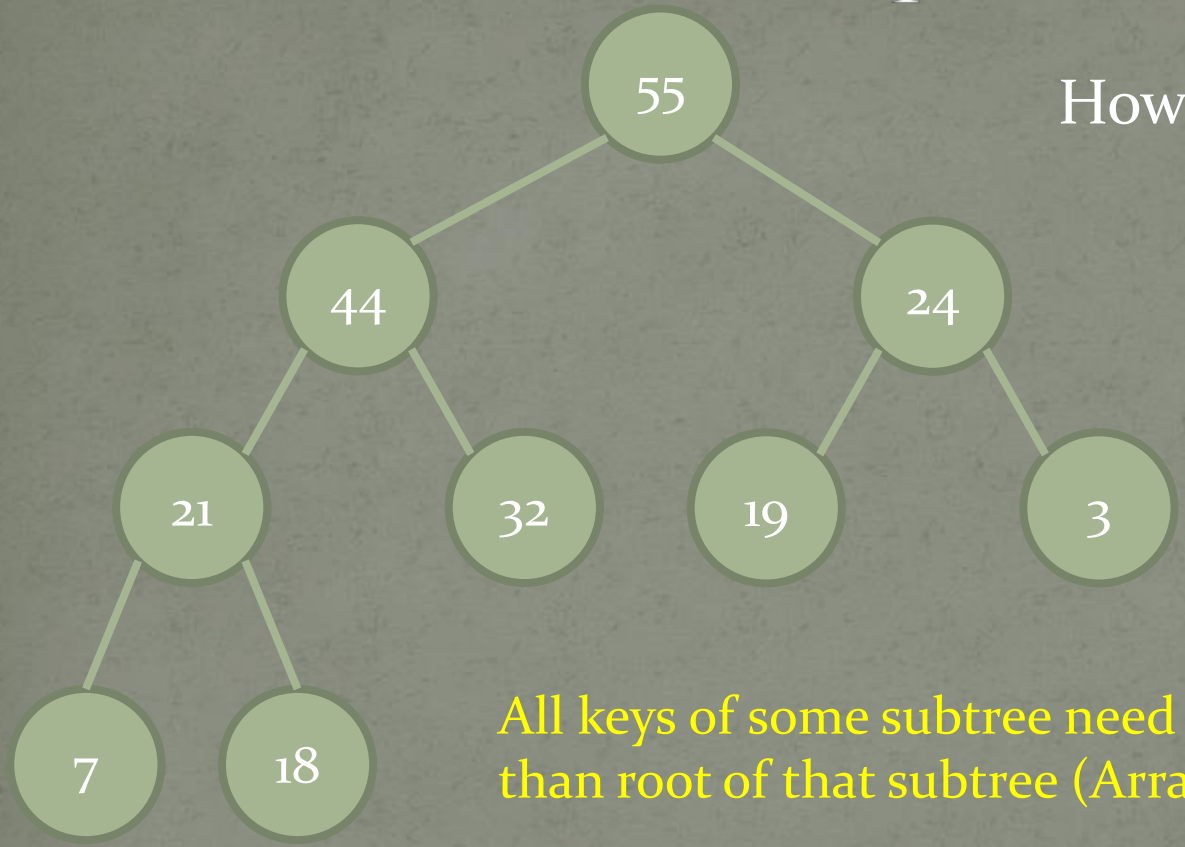
Algorithms and Data Structures

Andrzej Pisarski

Plan of the lecture

- Heap (~~sterta~~, kopiec; español: montículo)
  - Introduction
  - Example of implementations of heap
  - Heap based on tree
  - Heapsort

Introduction – Heap max



How to obtain index of:

Father = floor(i/2)
Left child = 2*i
Right child = 2*i+1

All keys of some subtree need to have value not bigger than root of that subtree (Array[i]<=Array[Father[i]])

0	1	2	3	4	5	6	7	8	9	10	11
	55	44	24	21	32	19	3	7	18		

Example of implementations of heap

- Priority queues
  - types (max, min)
  - insertion (pushing) a new item
  - finding an item with max or min key
  - deleting (popping)

Heap based on tree

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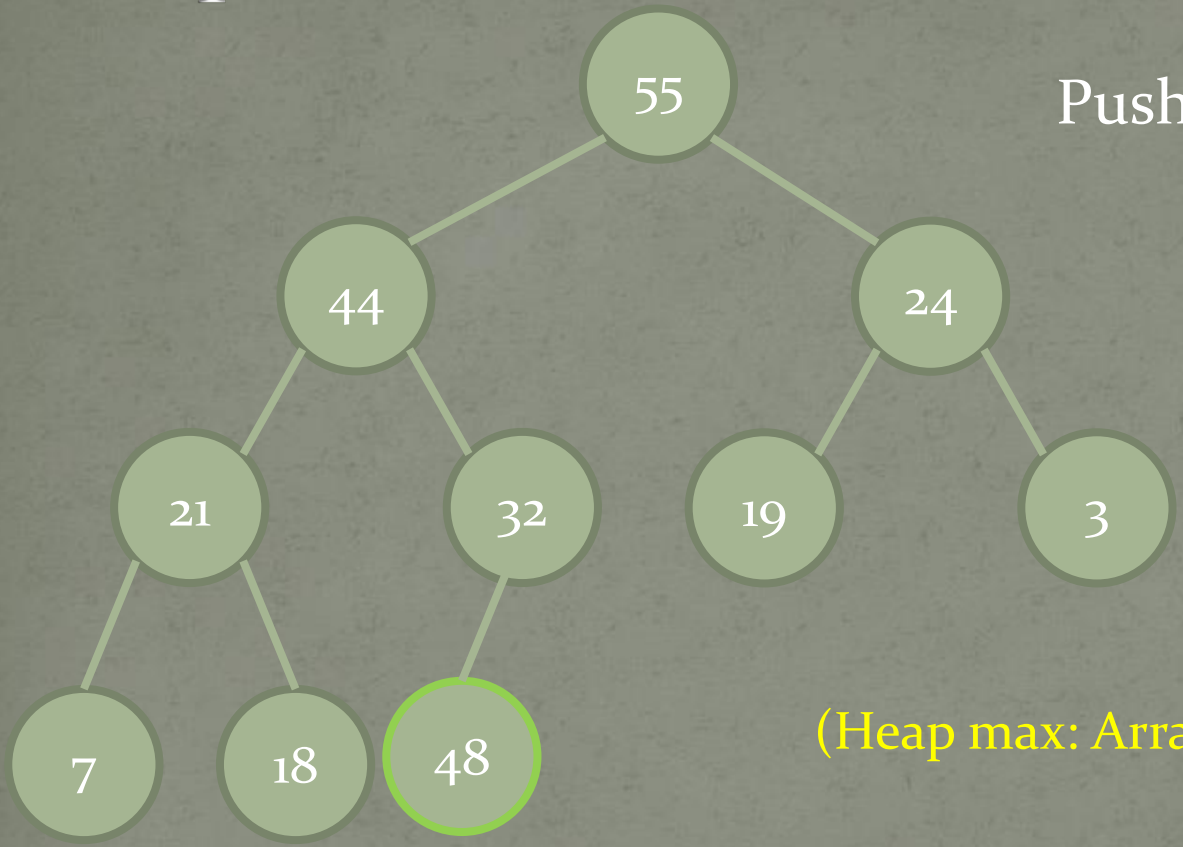

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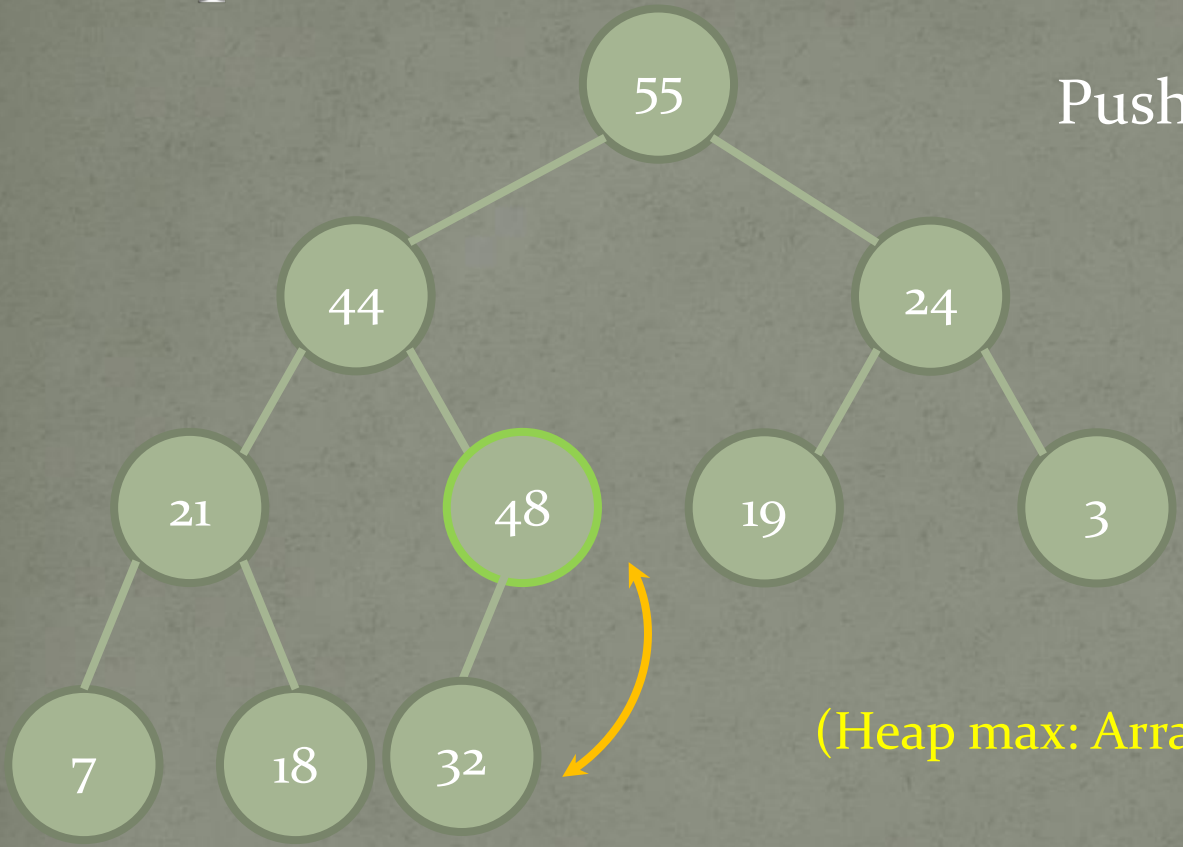

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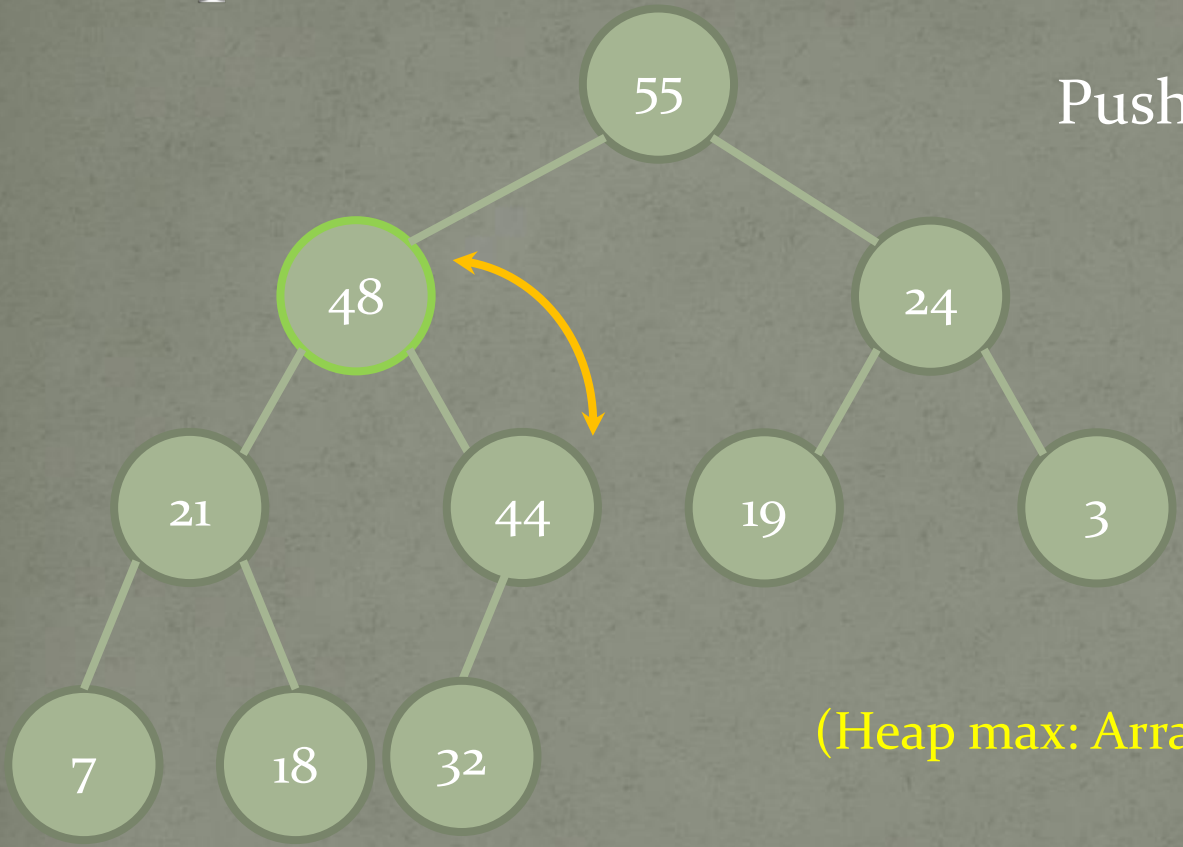

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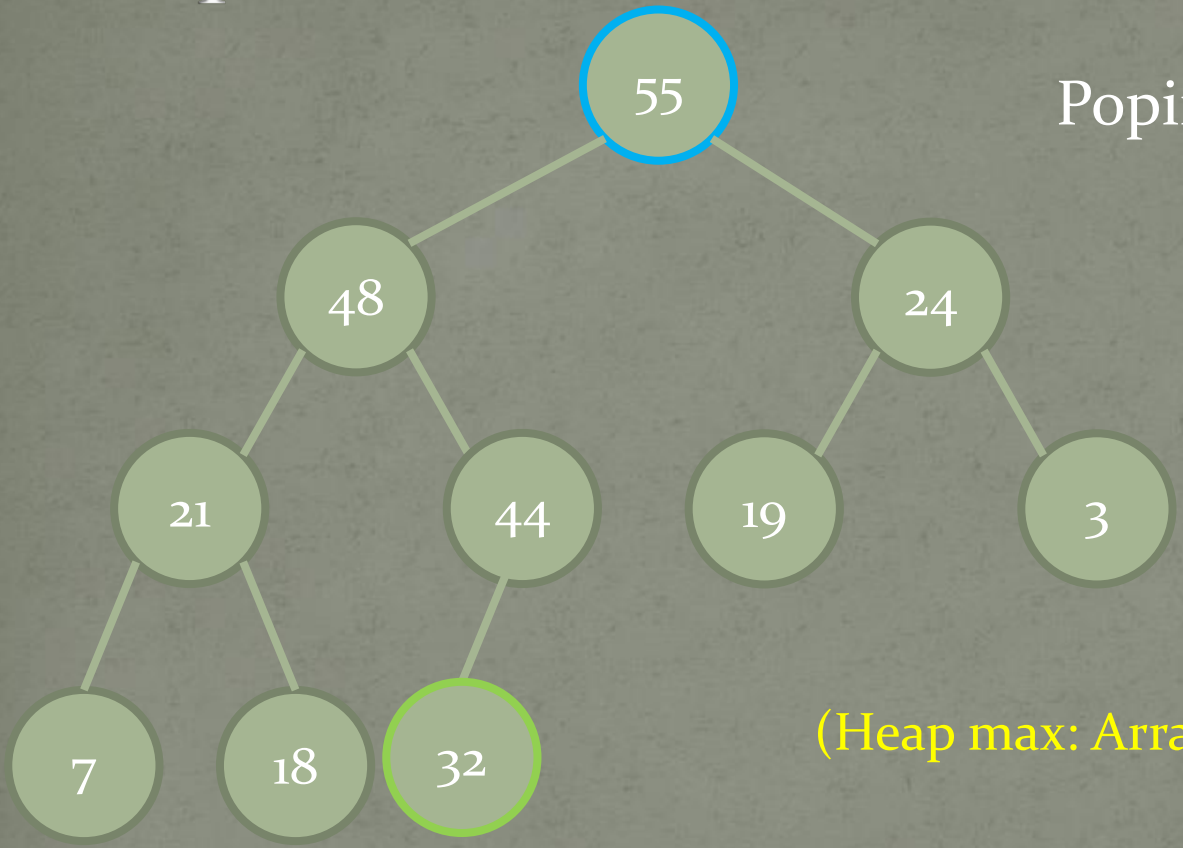

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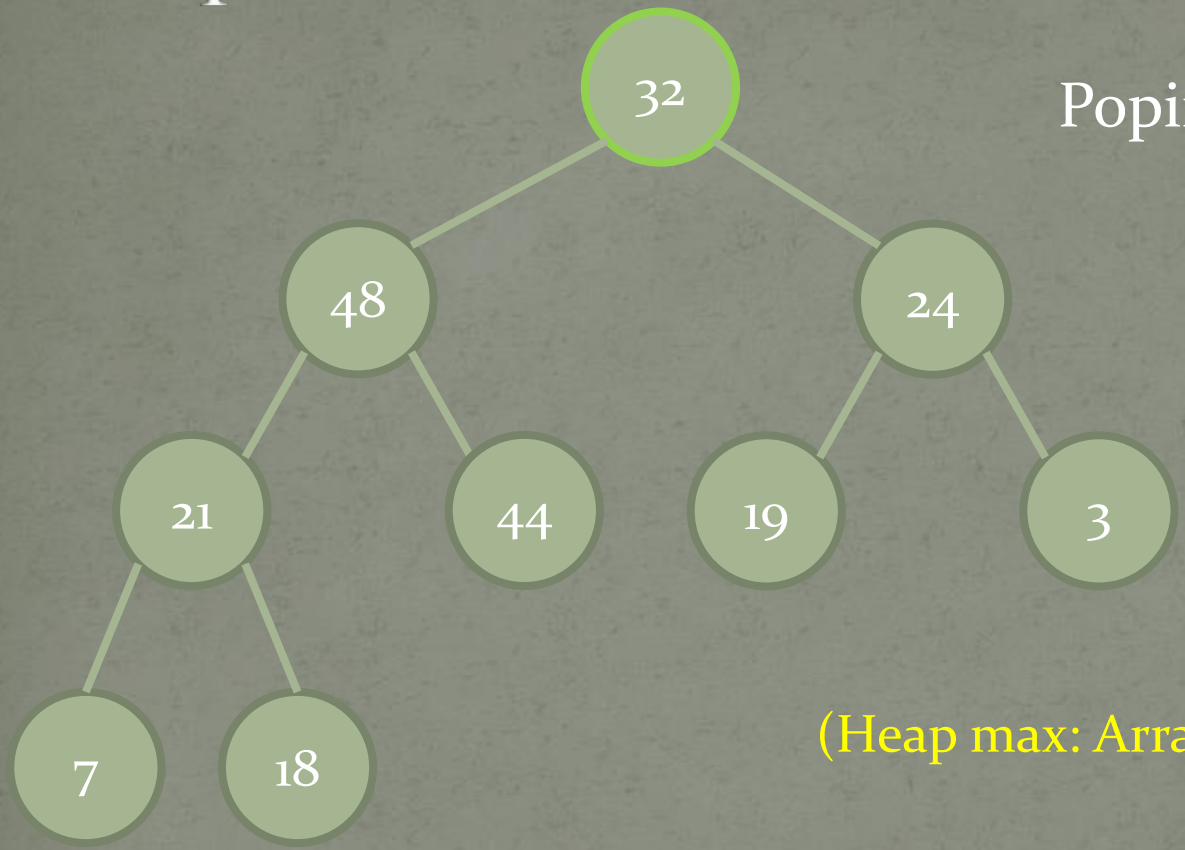

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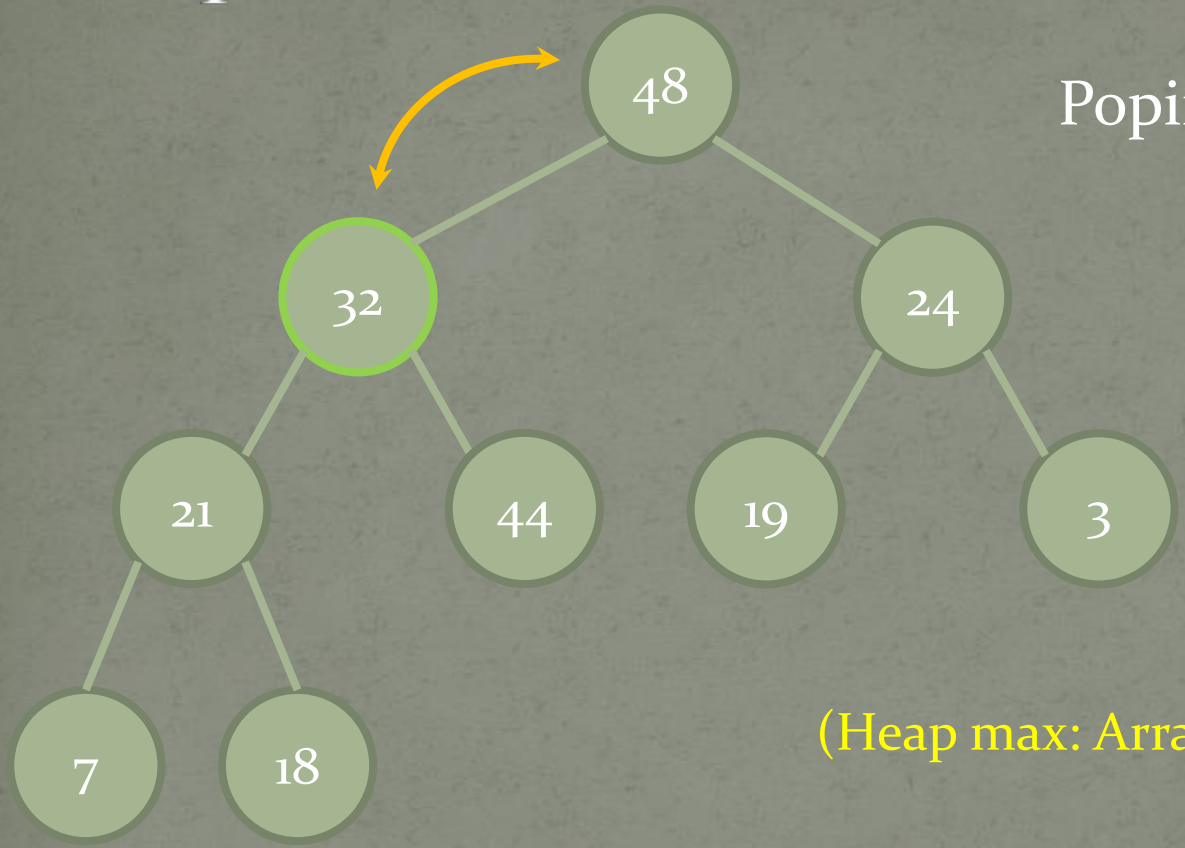

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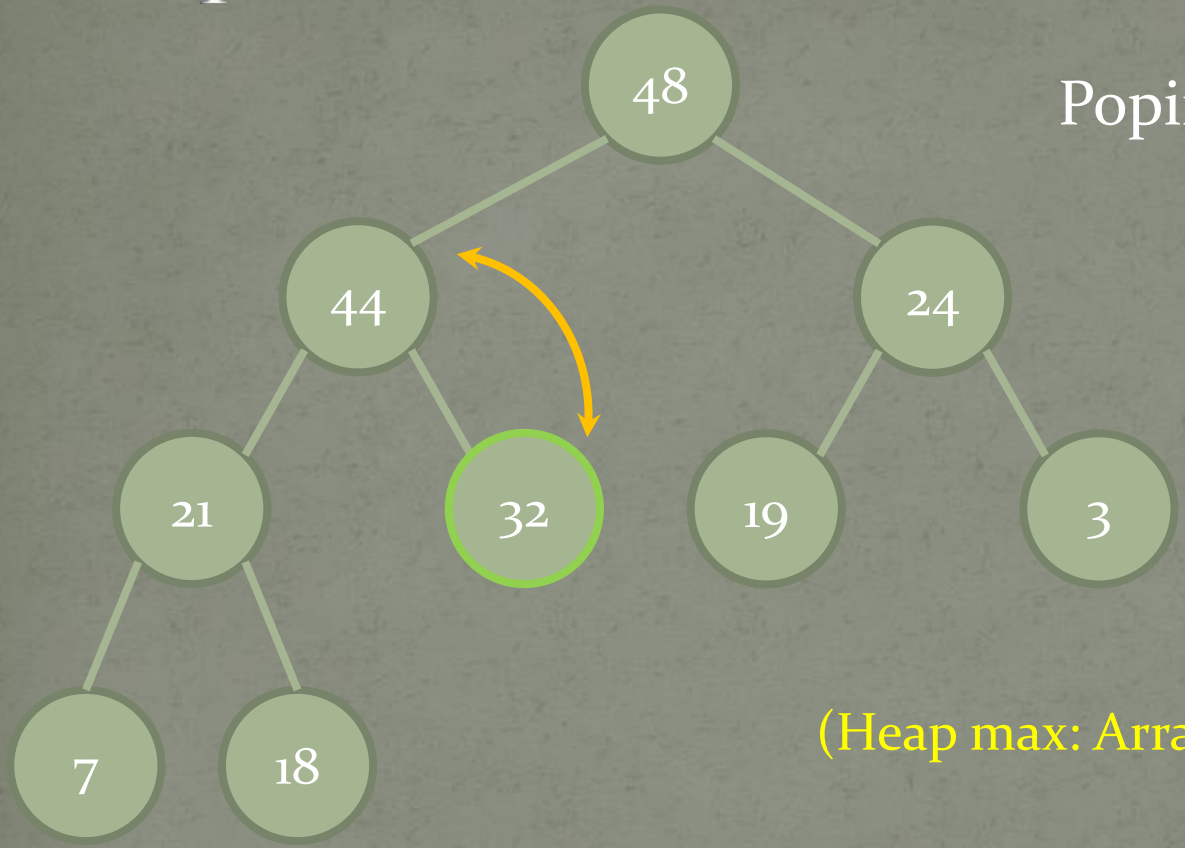

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Continue as long as size of the heap is not less than 2

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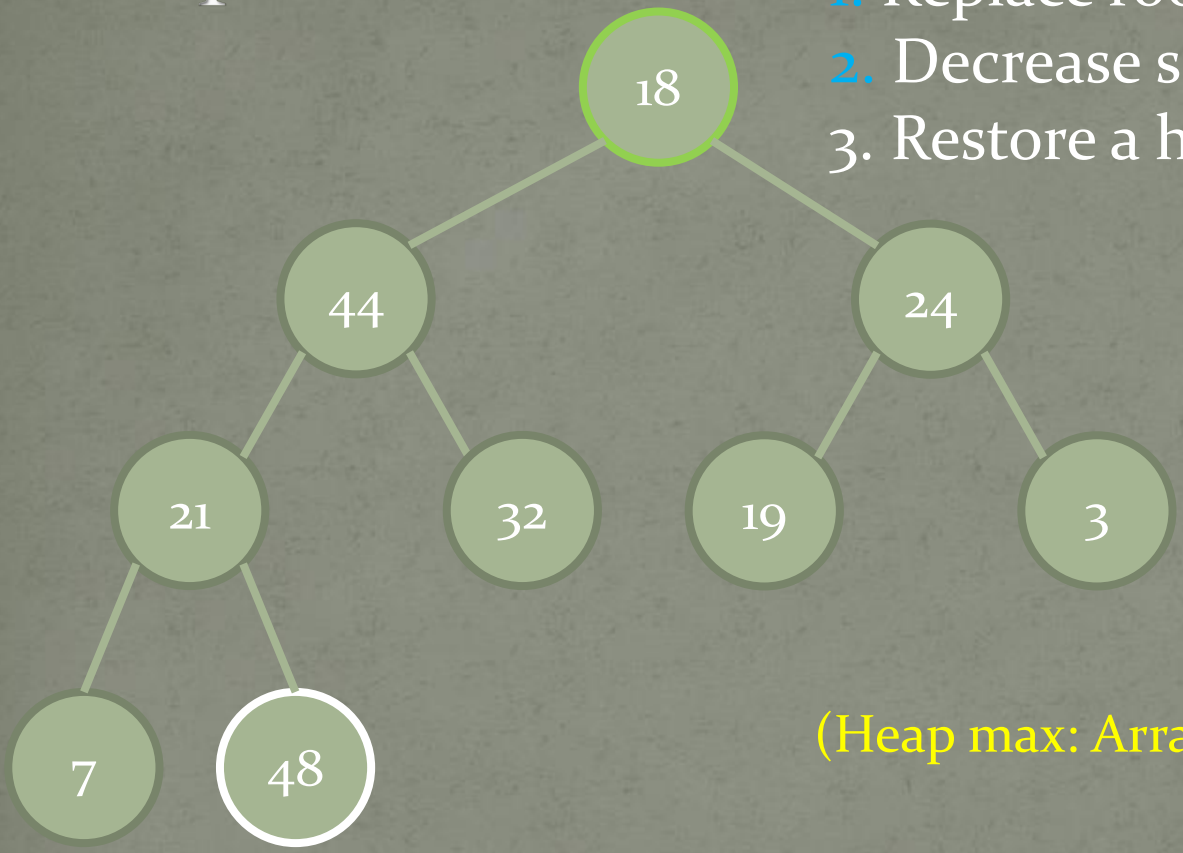

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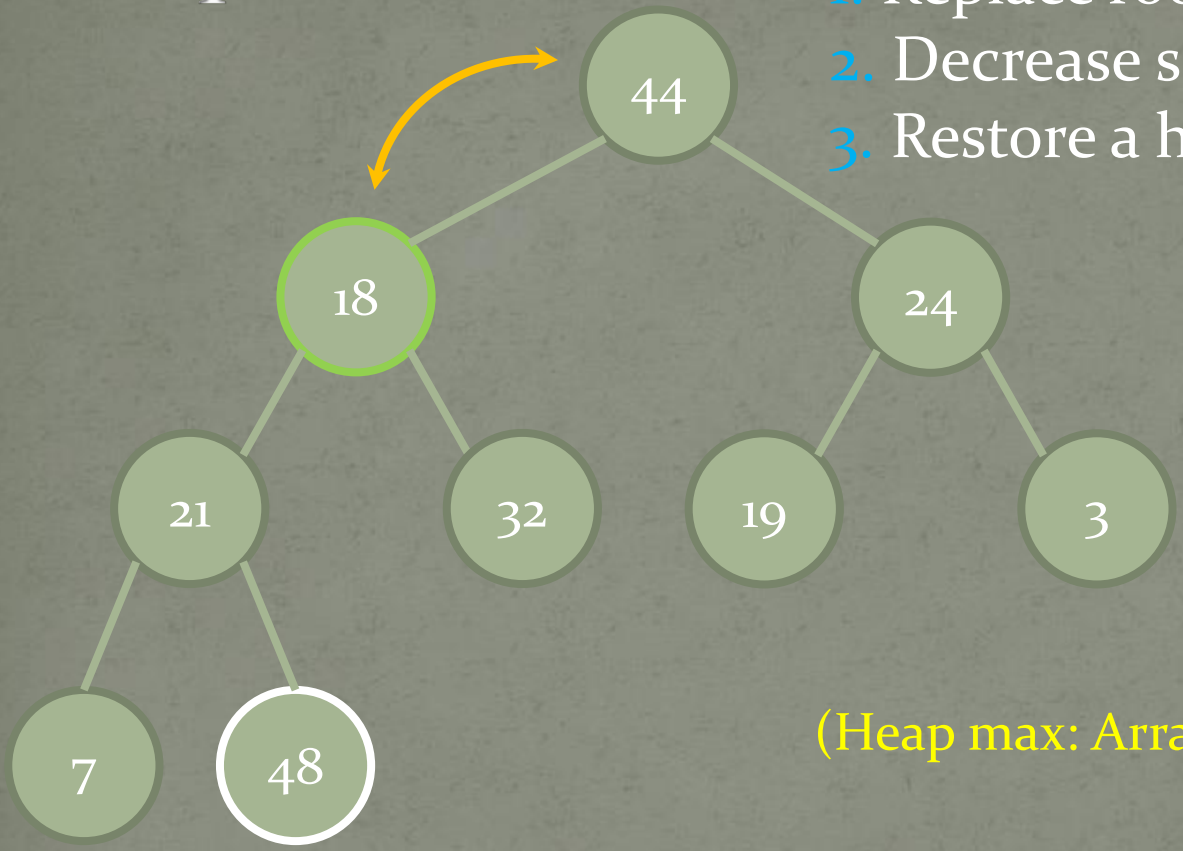

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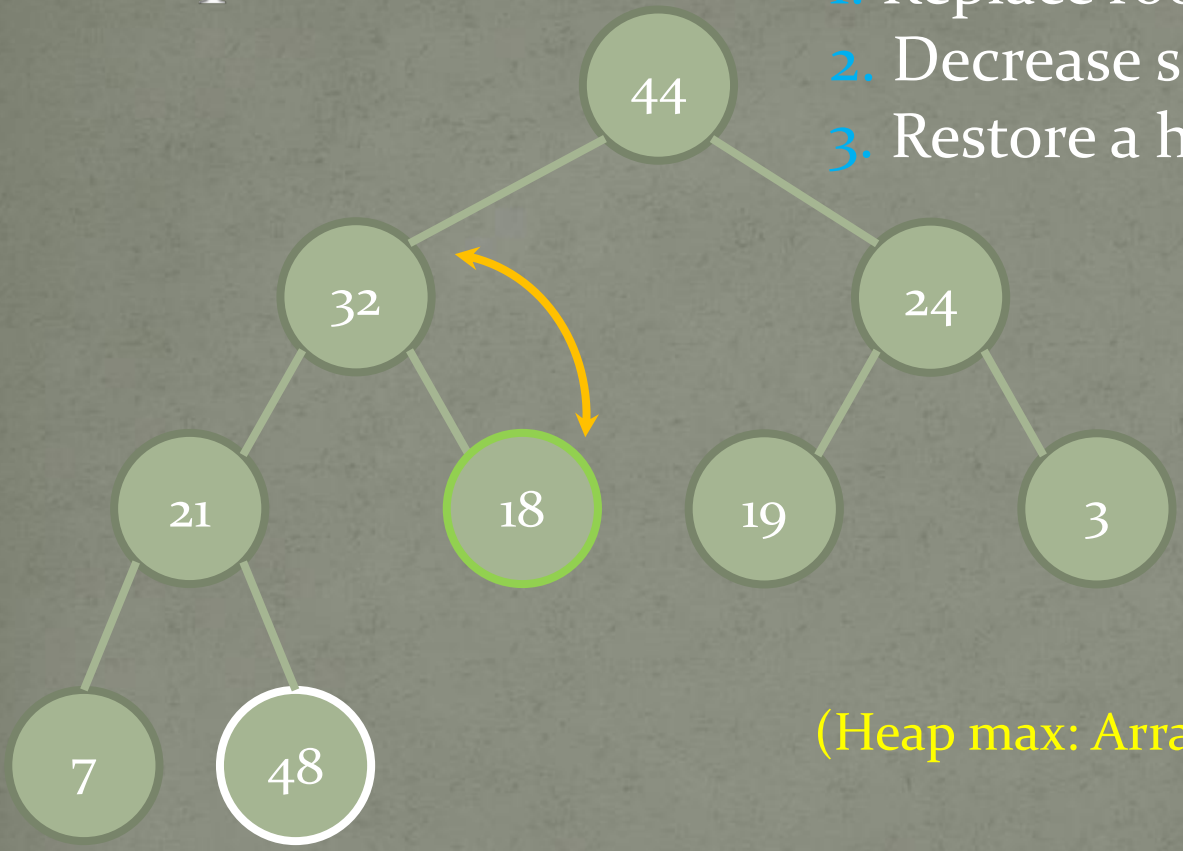

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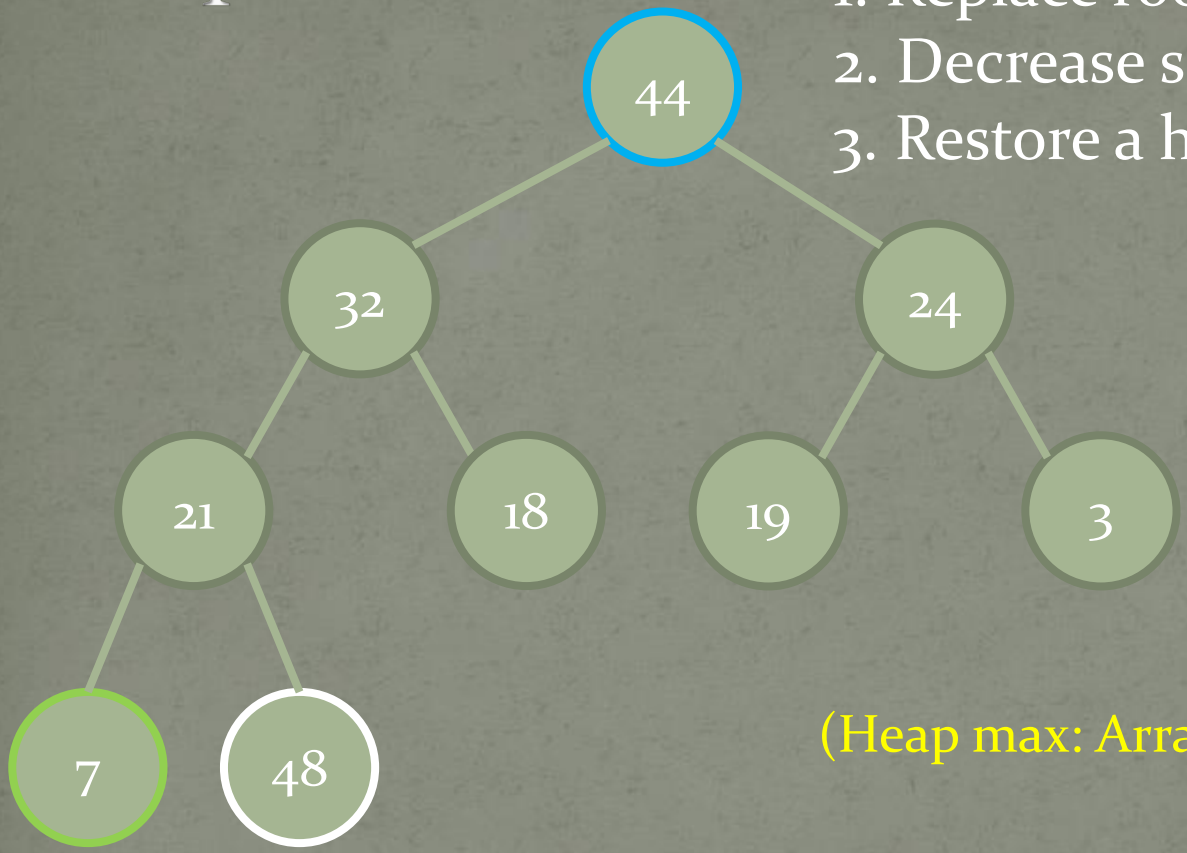

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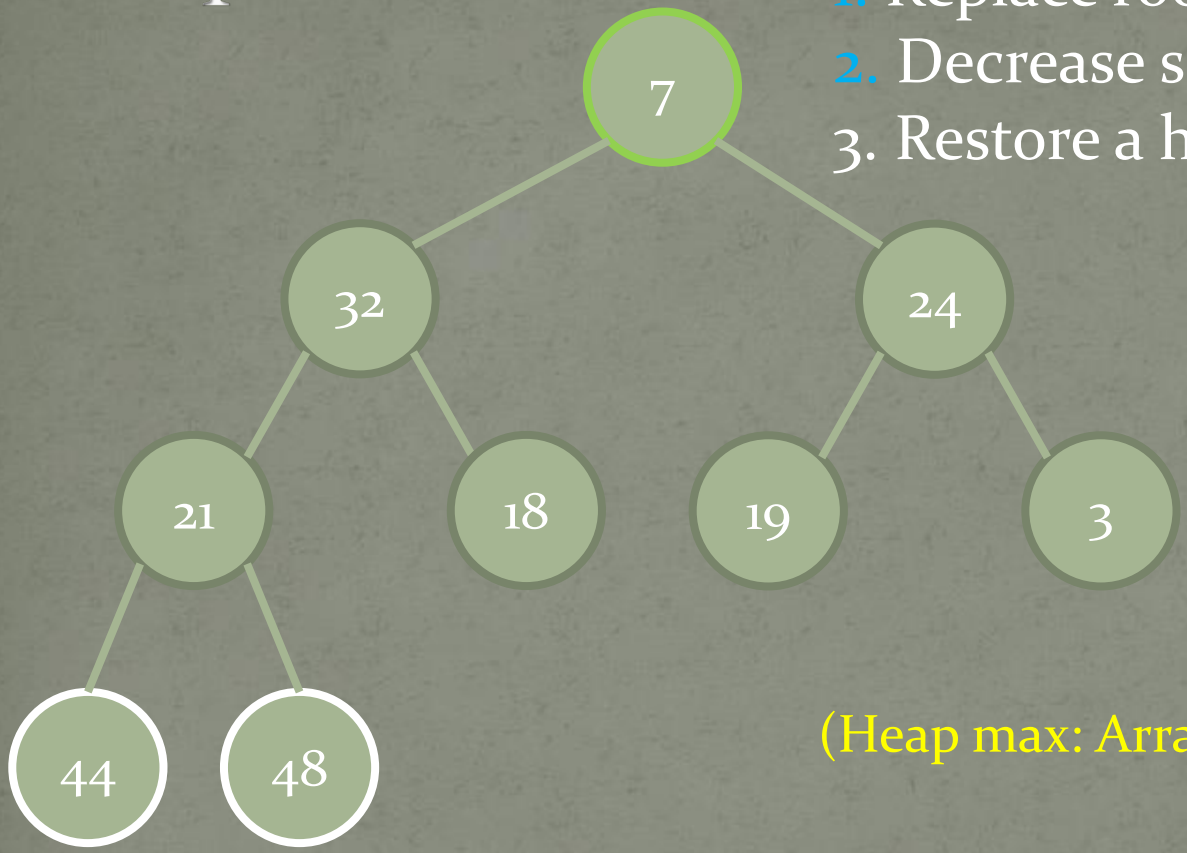

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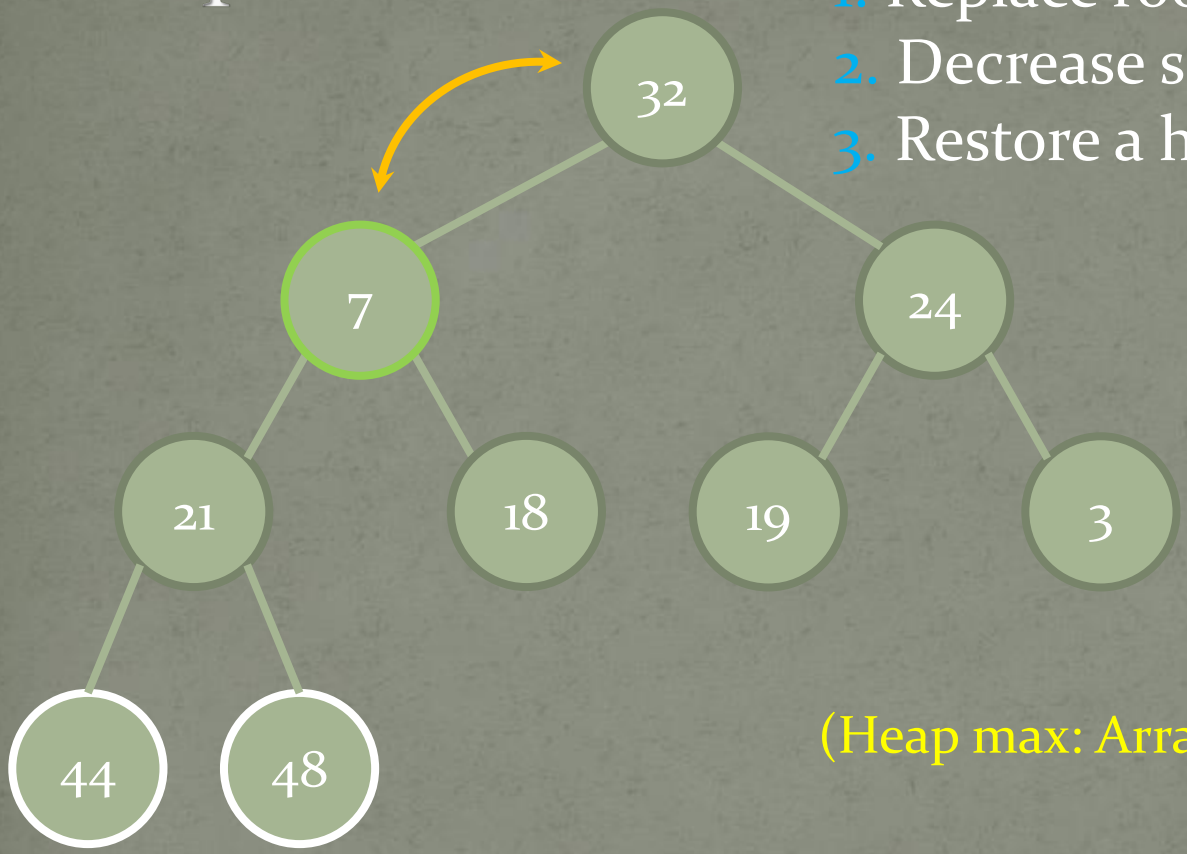

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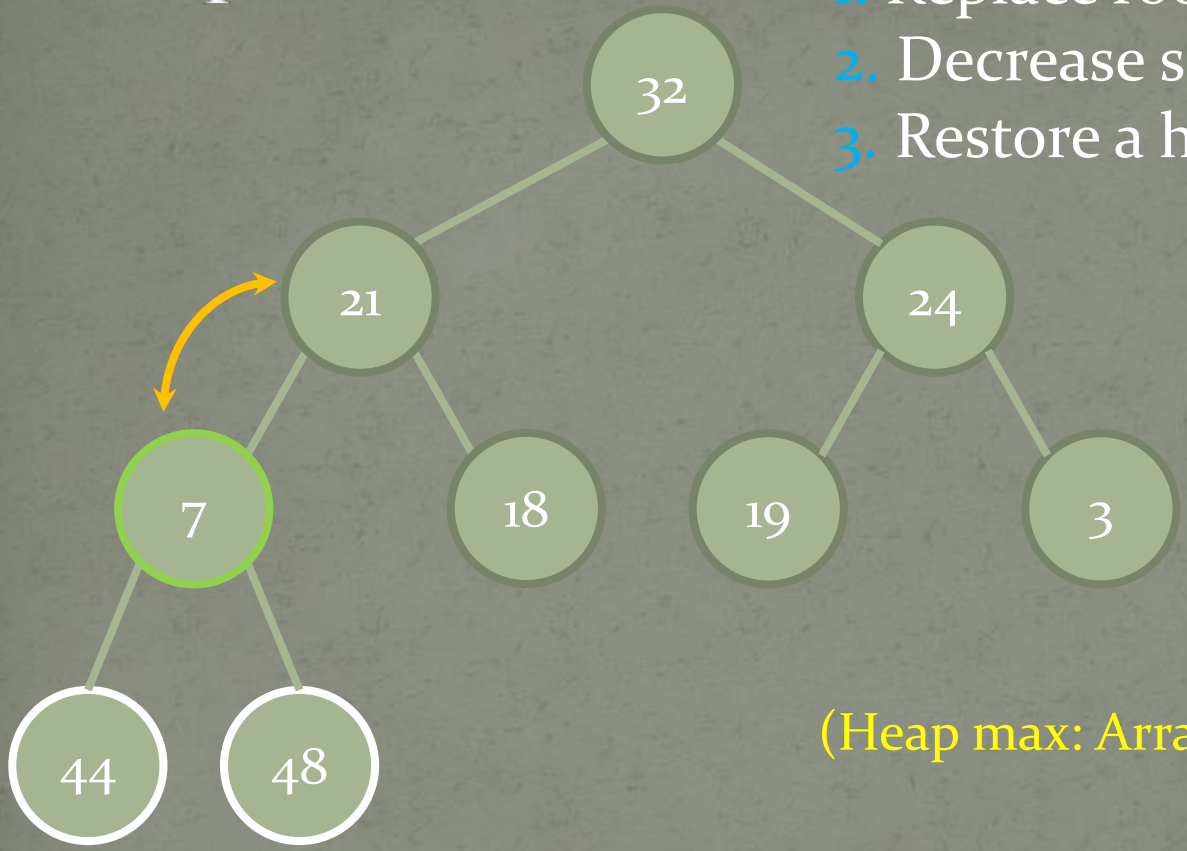

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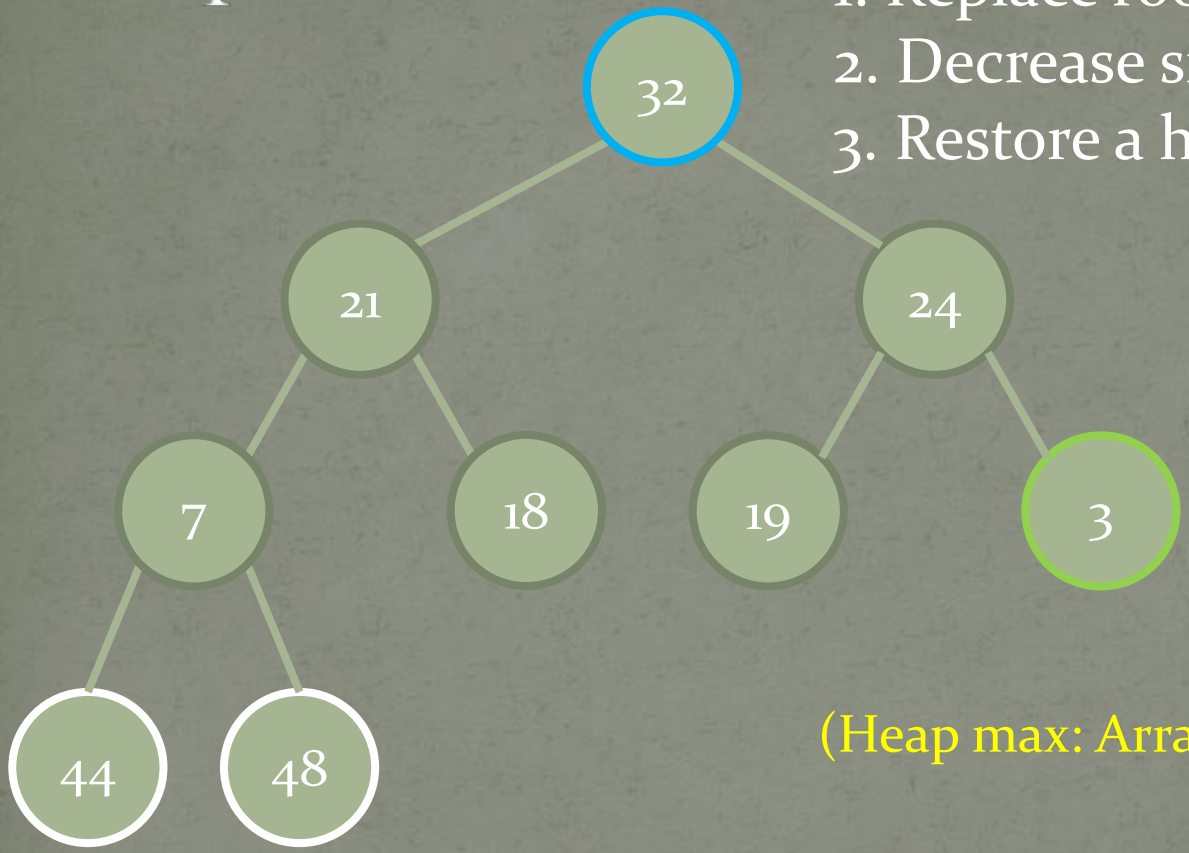

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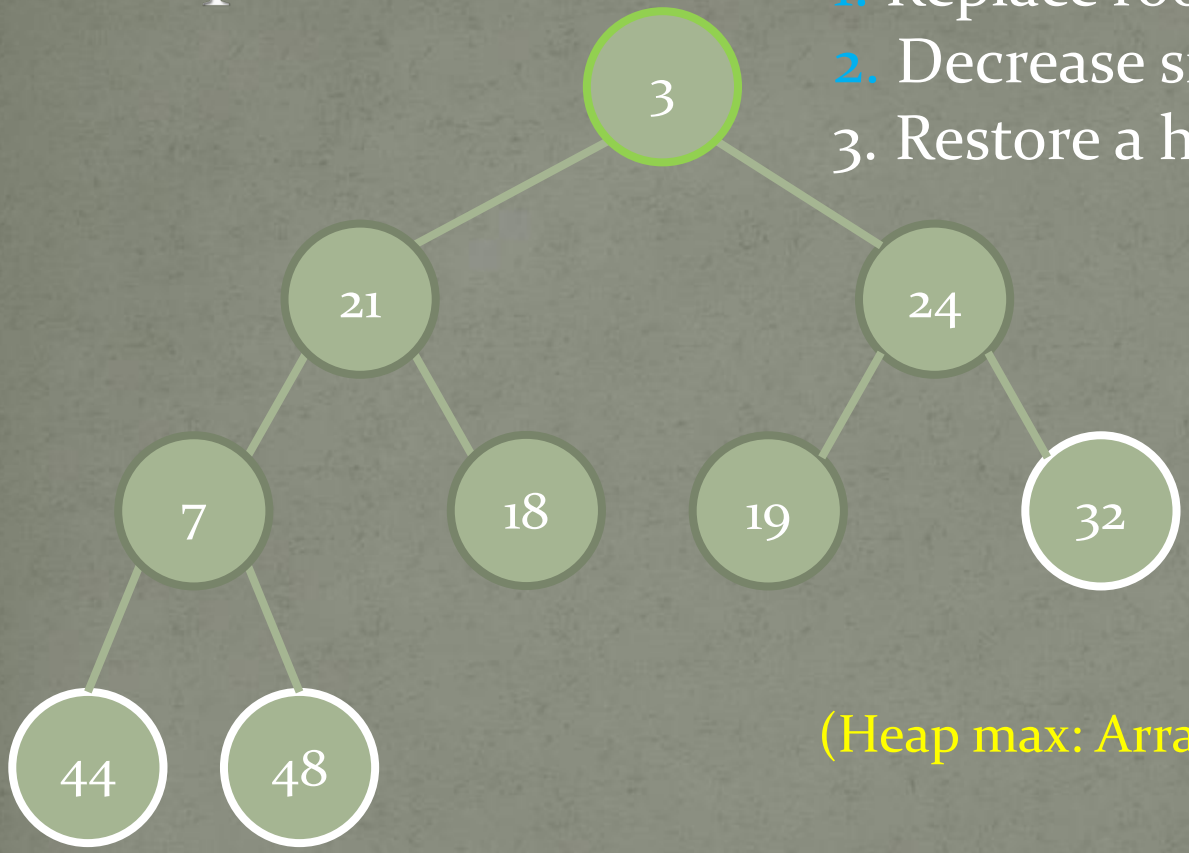

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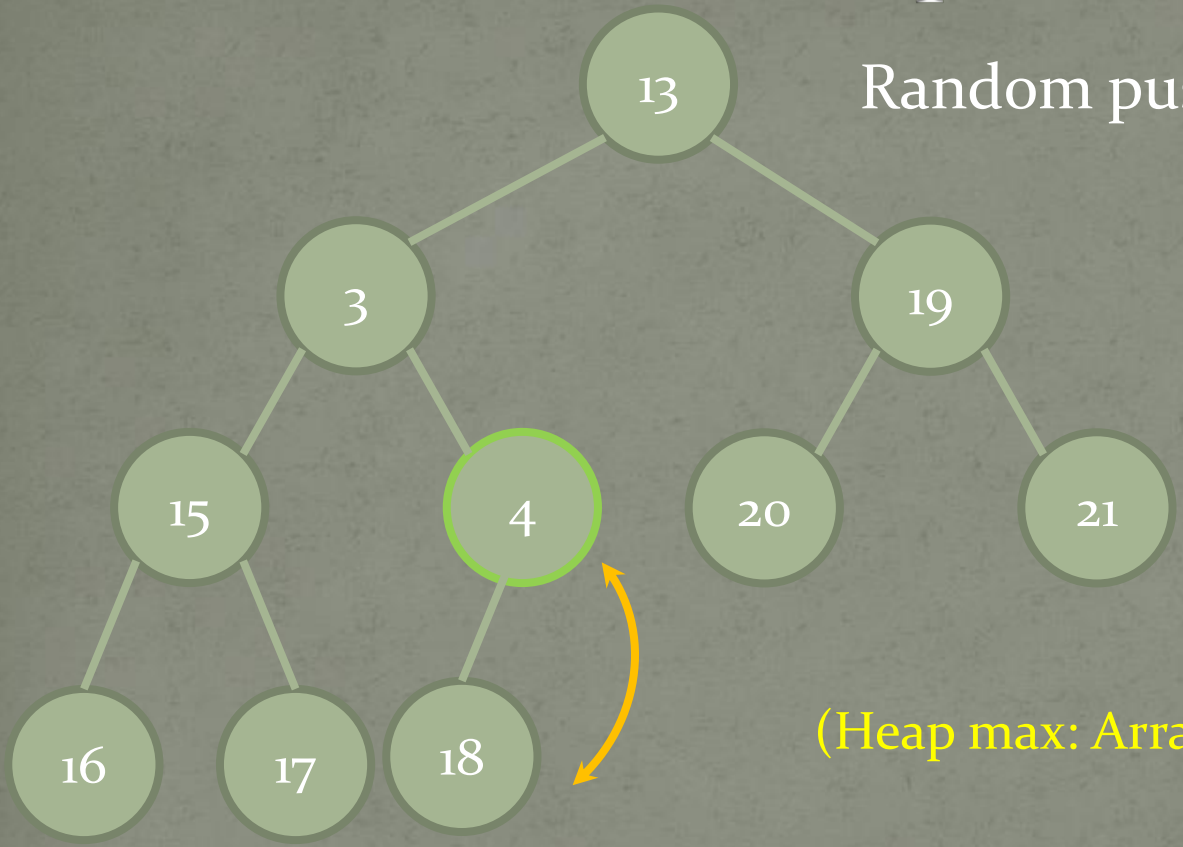

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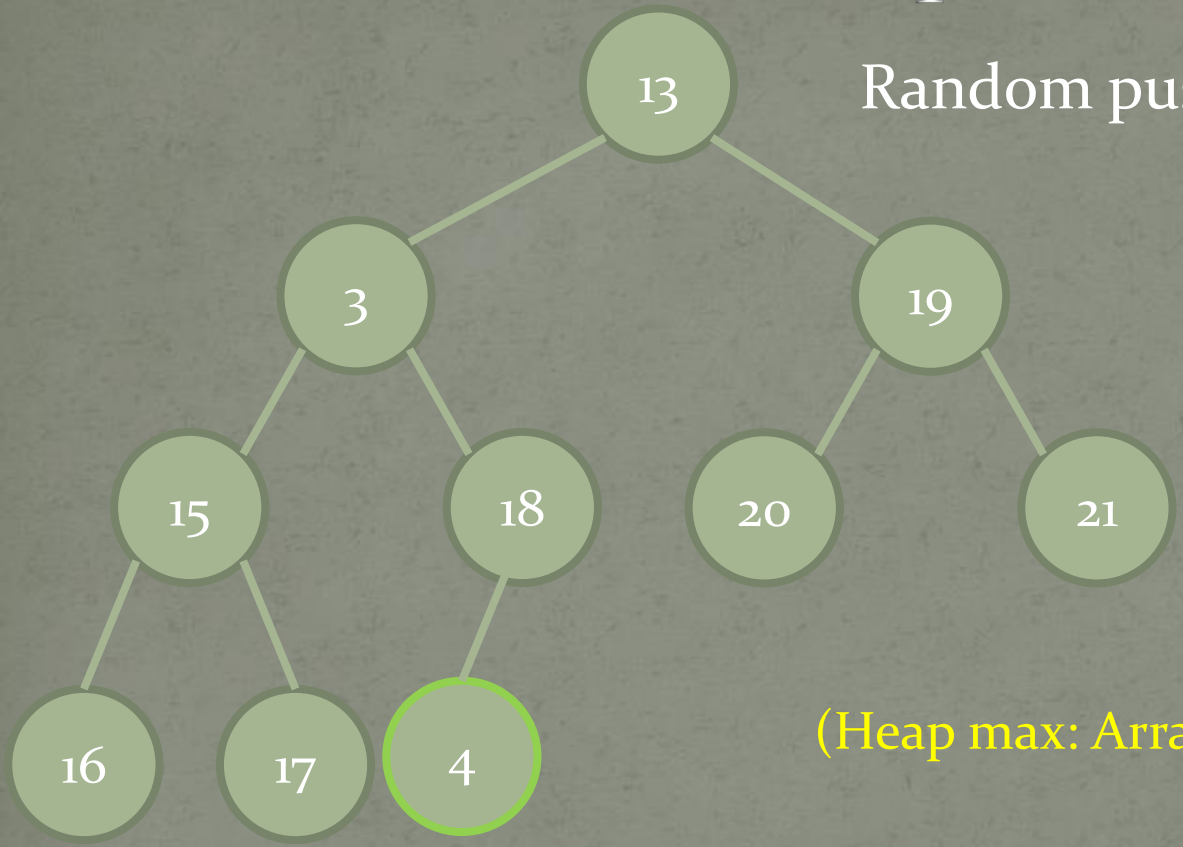

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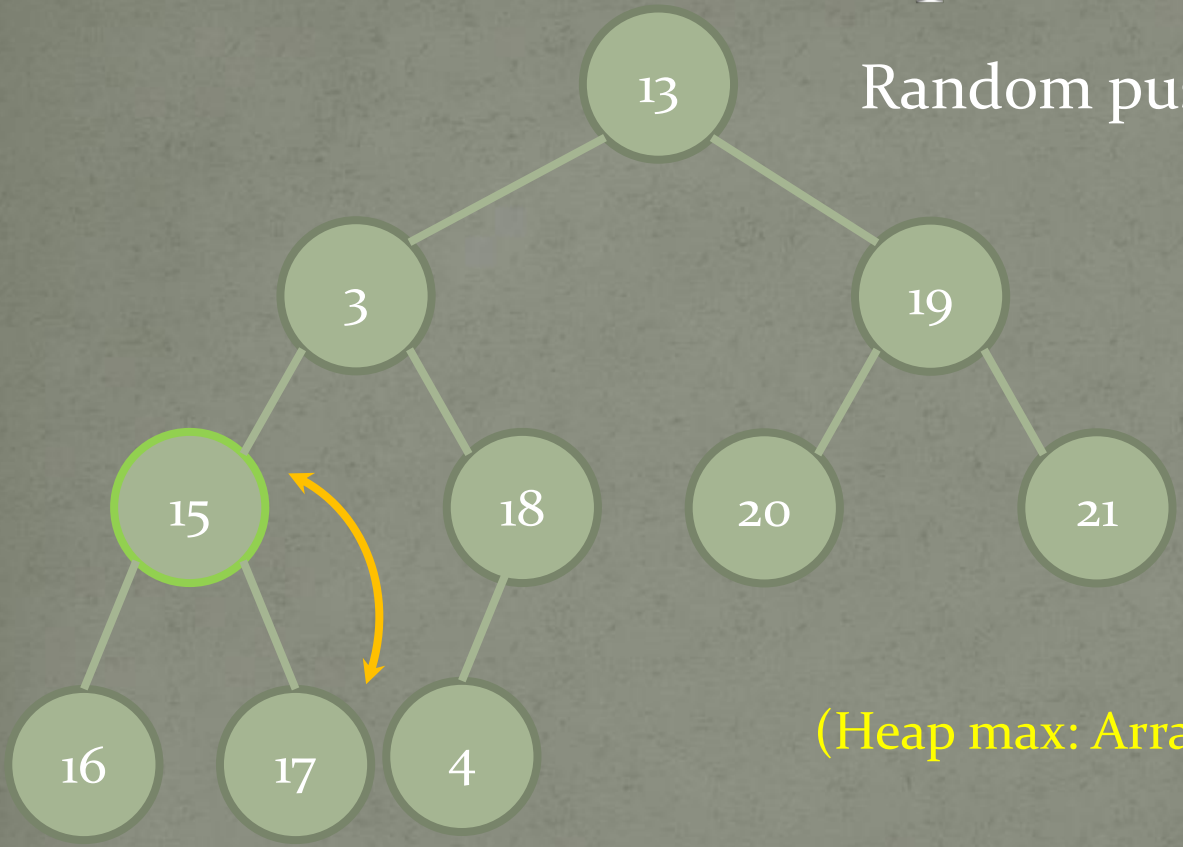

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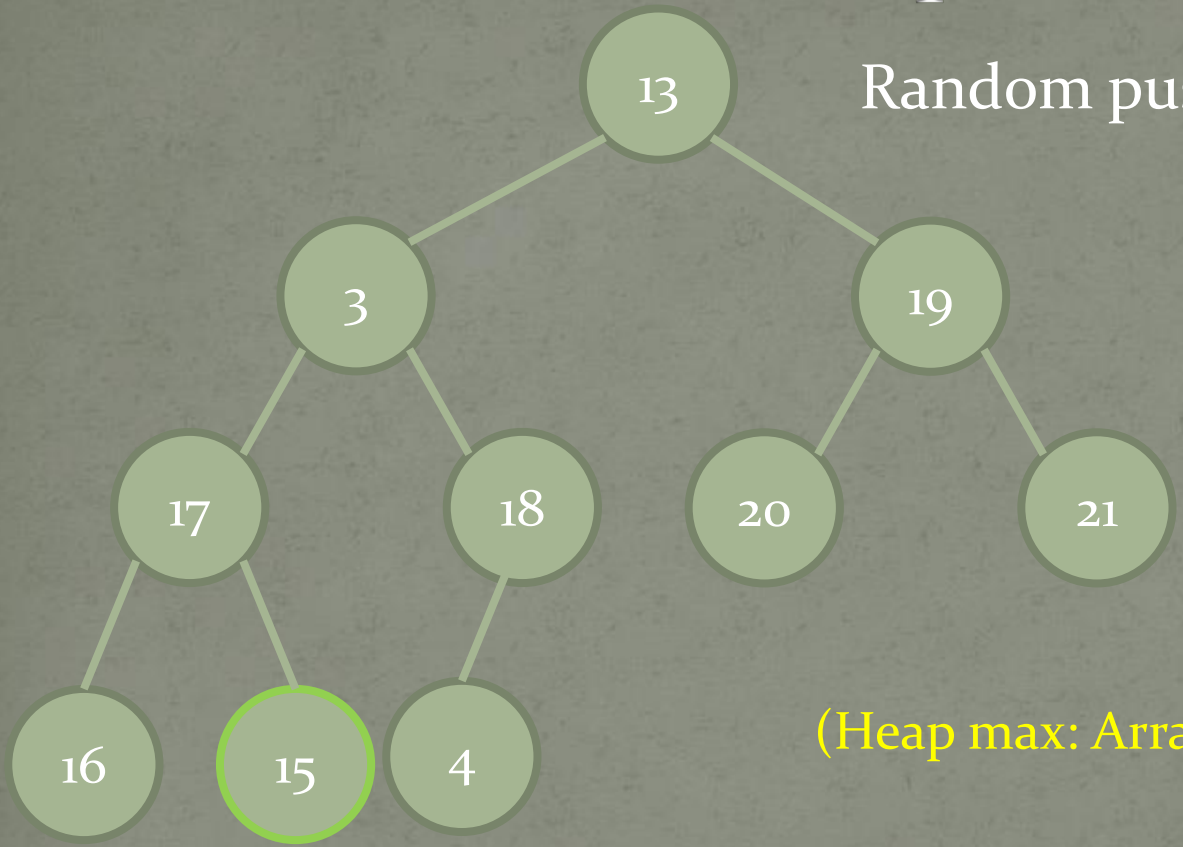

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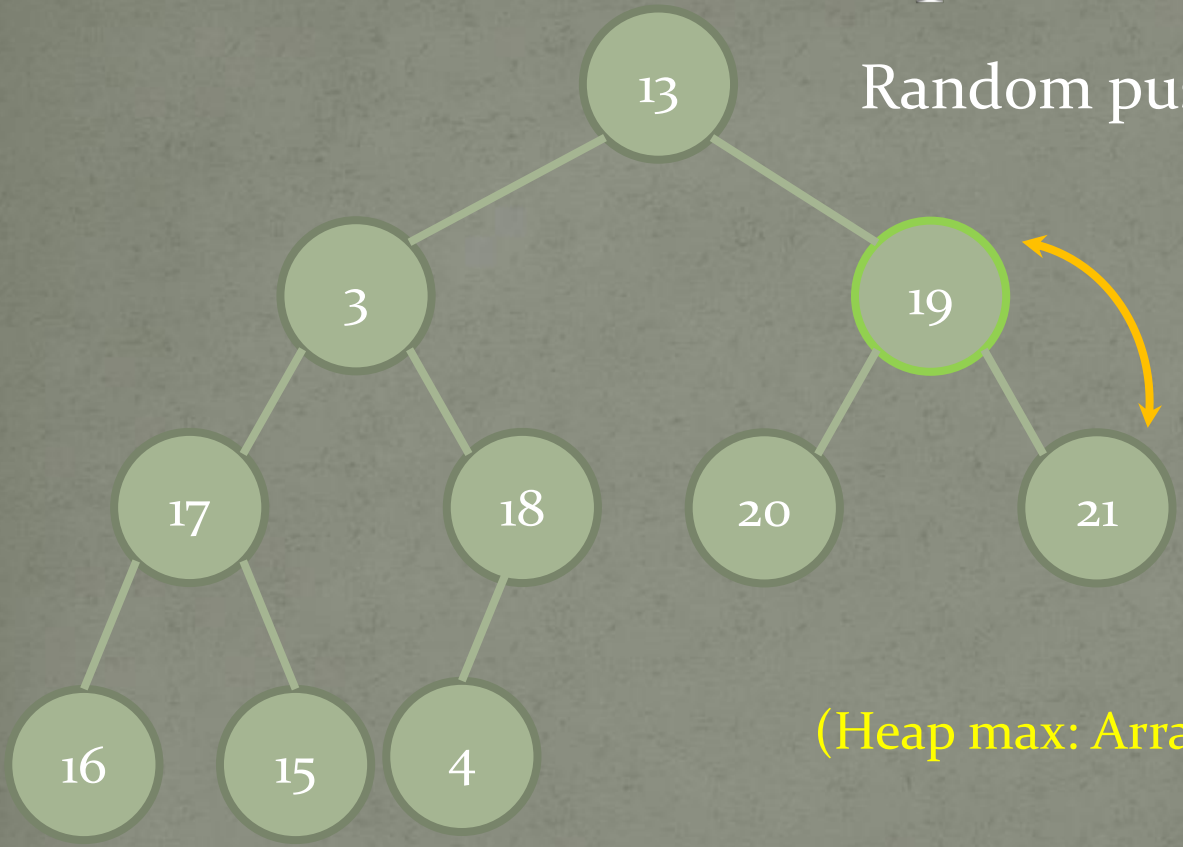

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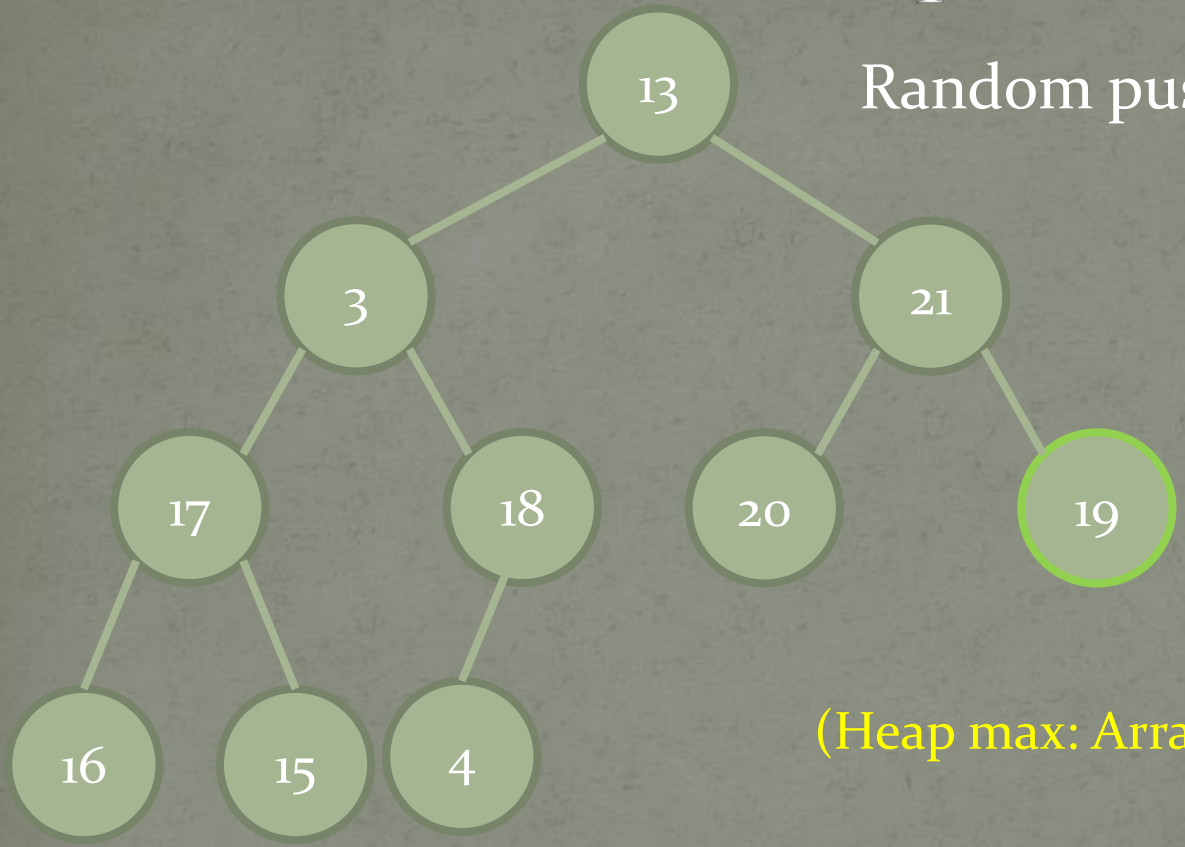

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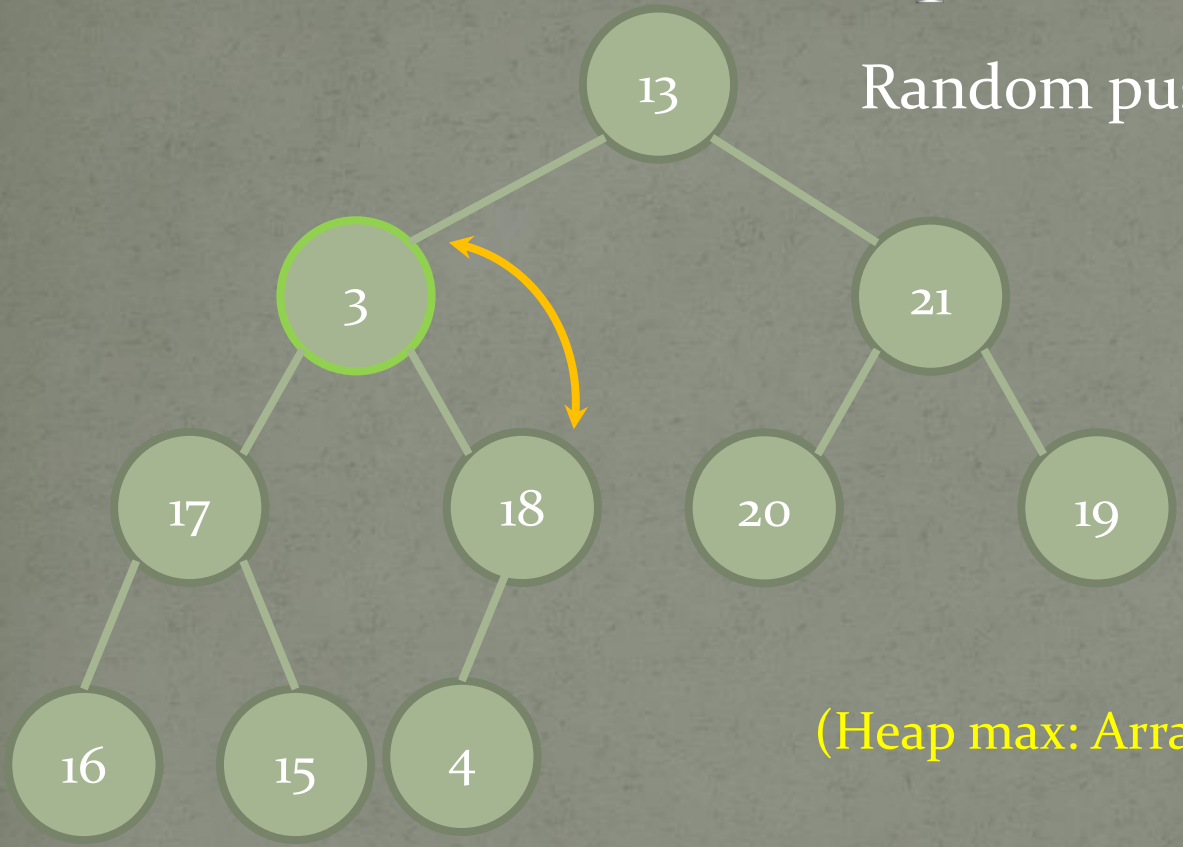

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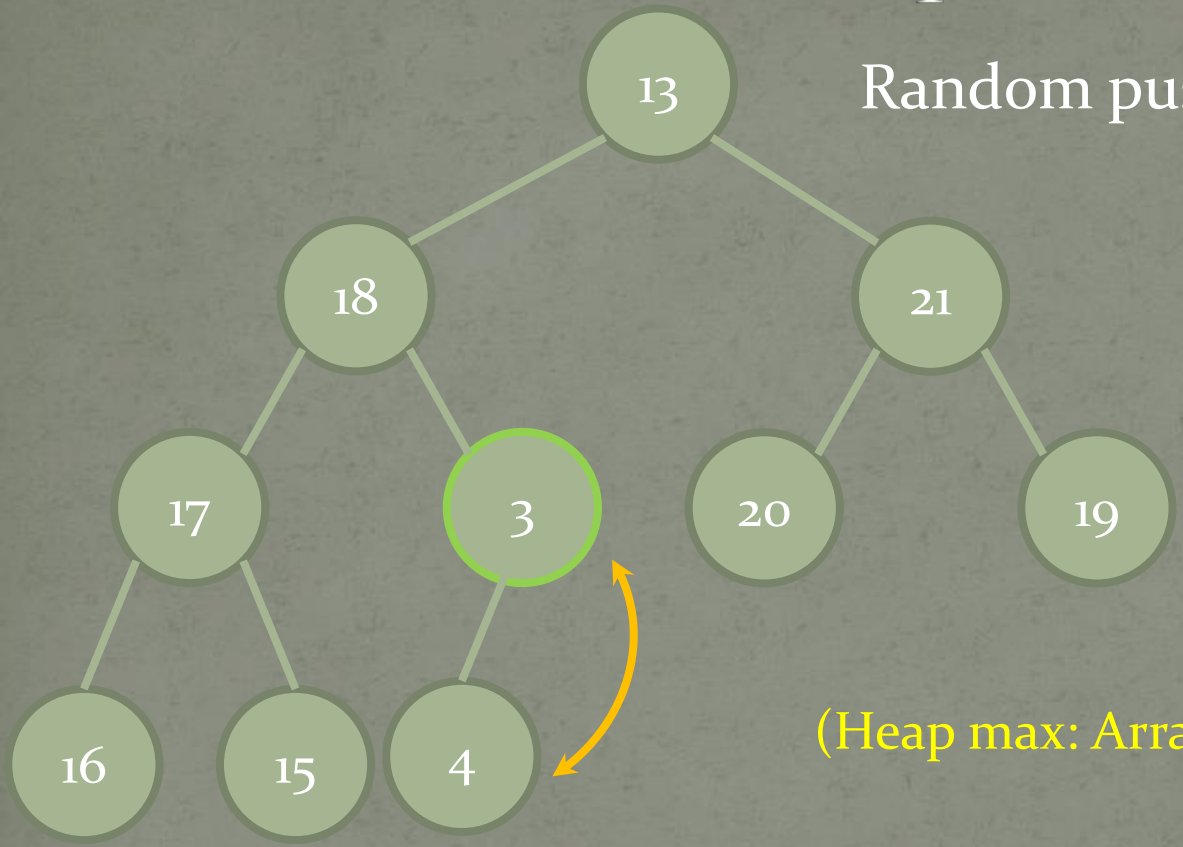

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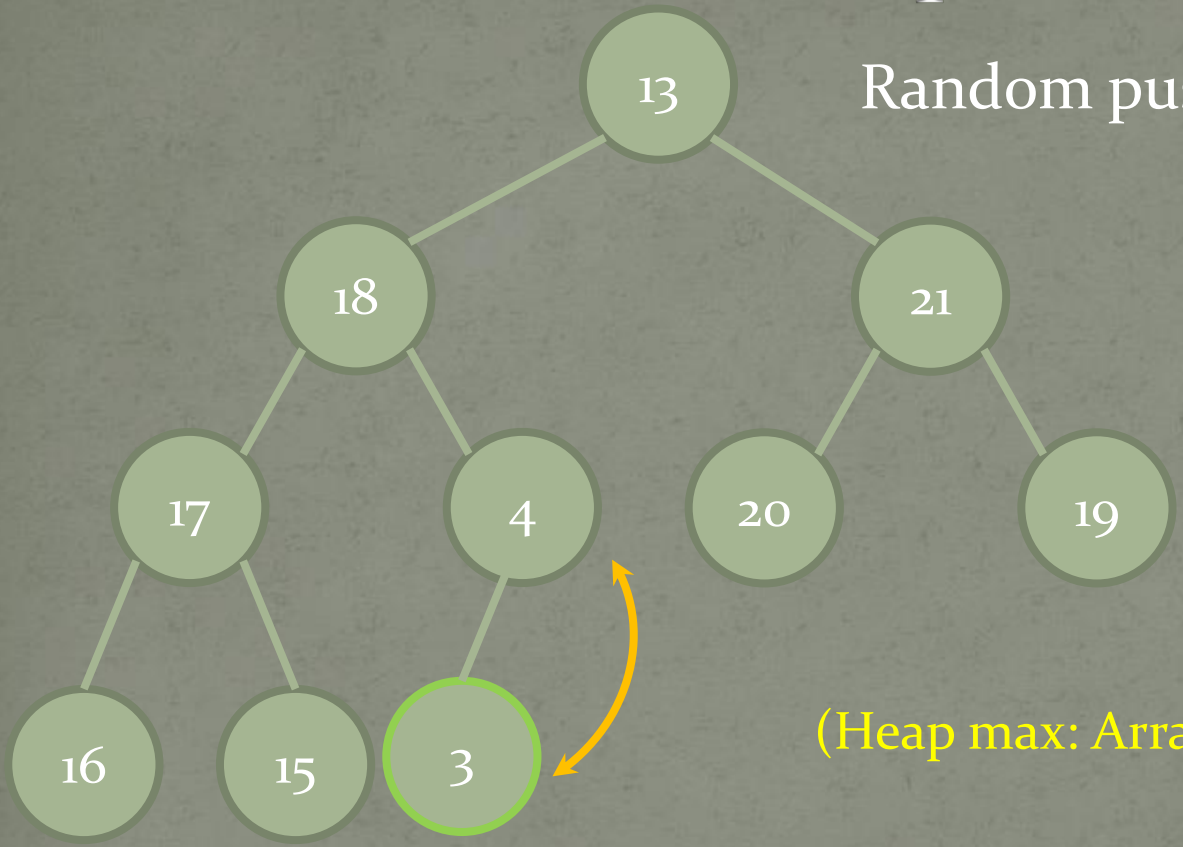

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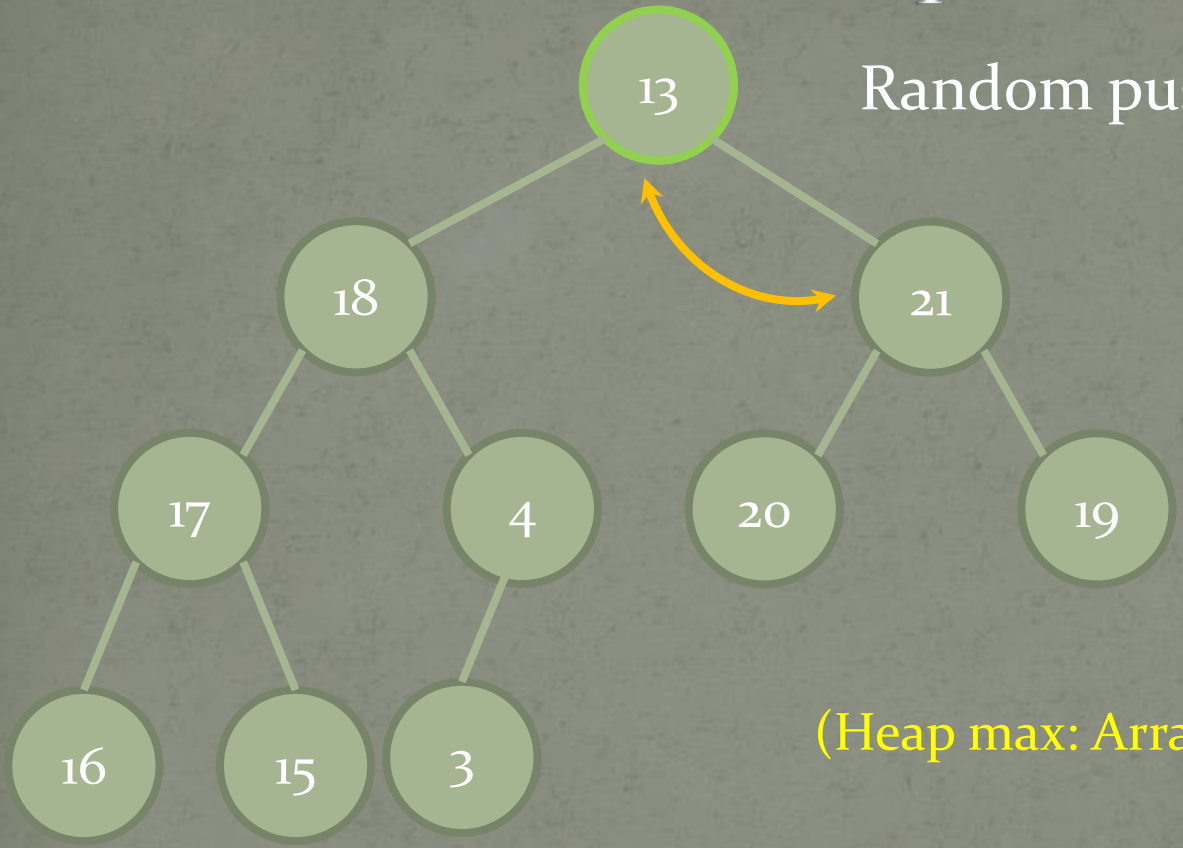

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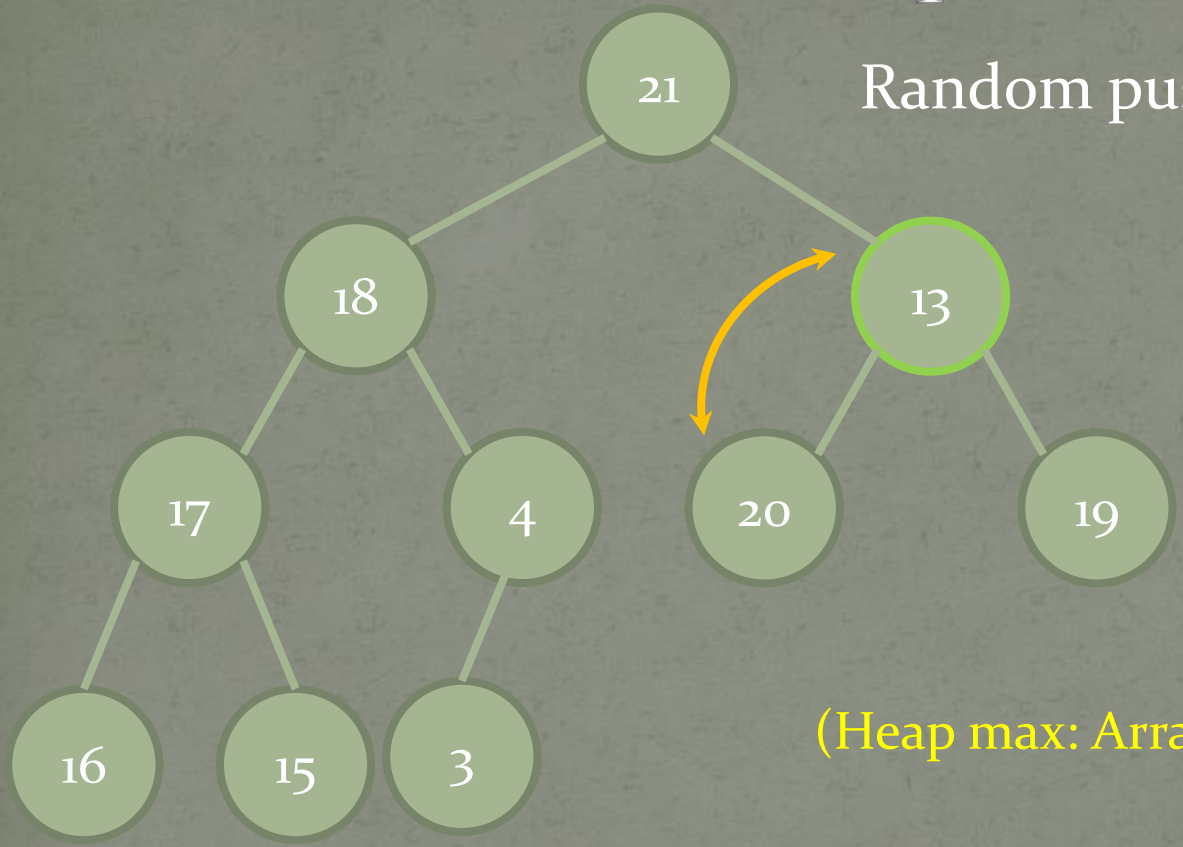

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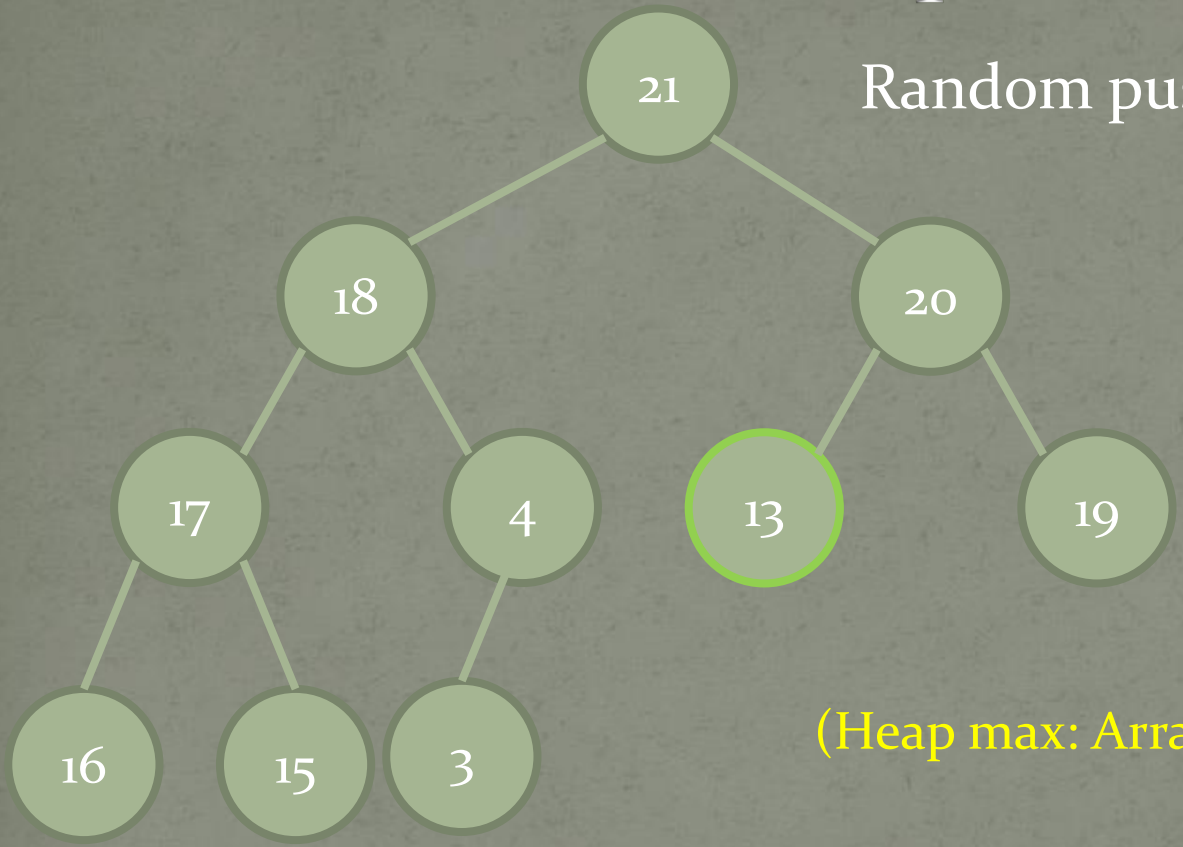

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