

Algorithms and Data Structures

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Plan of the lecture

1. Hash Tables
2. Hashing with Linear Probing
3. Hashing with Quadratic Probing
4. Hashing with Double Hashing
5. Hashing with Separate Chaining
6. Efficiency

Hash Tables

The basic idea:

Take a record (**key**) and convert it using **some function** to numeric value (**hash key**)
some function = **hash function**

Example of hash function (division method):

$$\text{hash key (index)} = \text{key} \% \text{number of slots}$$

Hash Tables

Example of hash function (division method):

hash key (index) = key % number of slots

0	1	2	3	4	5	6	7	8
81	-1	20	39	40	-1	-1	7	-1

$$0 = 81 \% 9$$

$$2 = 20 \% 9$$

$$3 = 39 \% 9$$

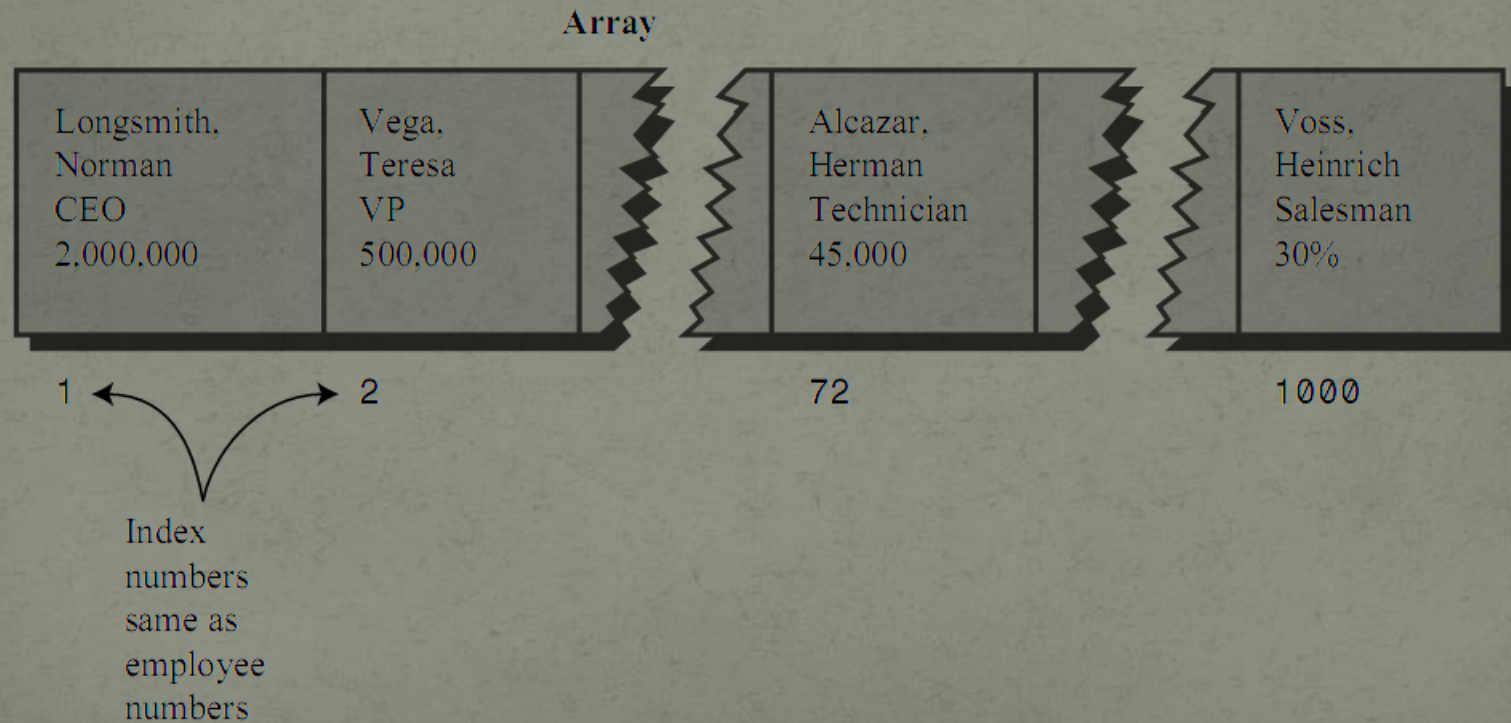
$$4 = 40 \% 9$$

$$7 = 7 \% 9$$

Hash Tables

Some other examples of keys:

- employee numbers



(example from R. Lafore book)

Hash Tables

Some other examples of keys:

- employee numbers,
- words (how to convert words to hash key?)

Sign	Code page ANSI ASCII		
	binary	decimal	aplhabet index
c	01100011	99	3
a	01100001	97	1
t	01110100	116	20
s	01110011	115	19

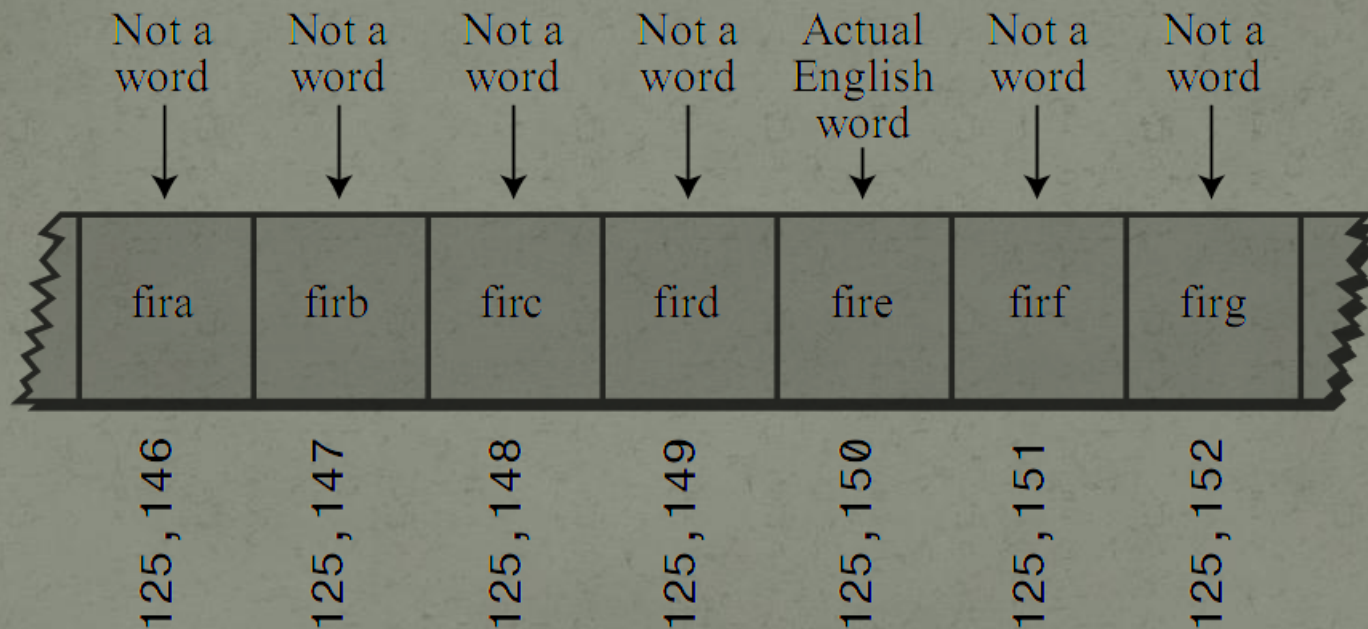
$$5467 = 5 * 10^3 + 4 * 10^2 + 6 * 10^1 + 7 * 10^0$$

$$\text{cats} = 3 * 27^3 + 1 * 27^2 + 20 * 27^1 + 19 * 27^0 = 60337$$

Hash Tables

Every potential word from „a” to „zzzzzzzzzz” (10 „z”)
require array size:

$$26^*27^9 + 26^*27^8 + \dots + 26^*27^1 + 26^*27^0 > 26^*27^9 = 198 * 10^{12}$$



(example from R. Lafore book)

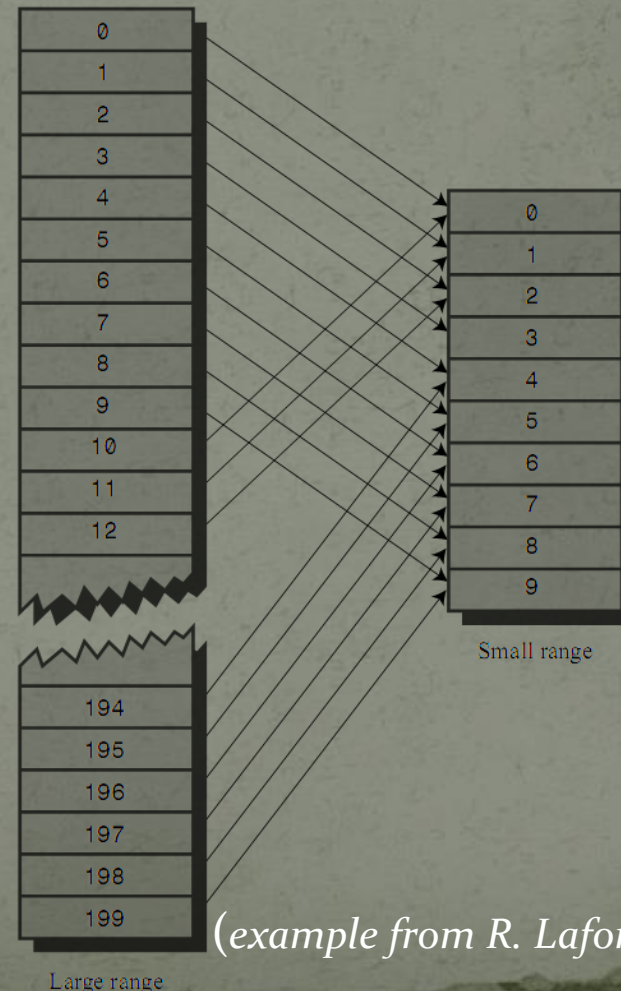
Hash Tables

$\text{hugeNumber} = 26 * 27^9 + 26 * 27^8 + \dots + 26 * 27^1 + 26 * 27^0$

$\text{arraySize} = \text{numberWords} * 2;$

$\text{arrayIndex} = \text{hugeNumber} \% \text{arraySize};$

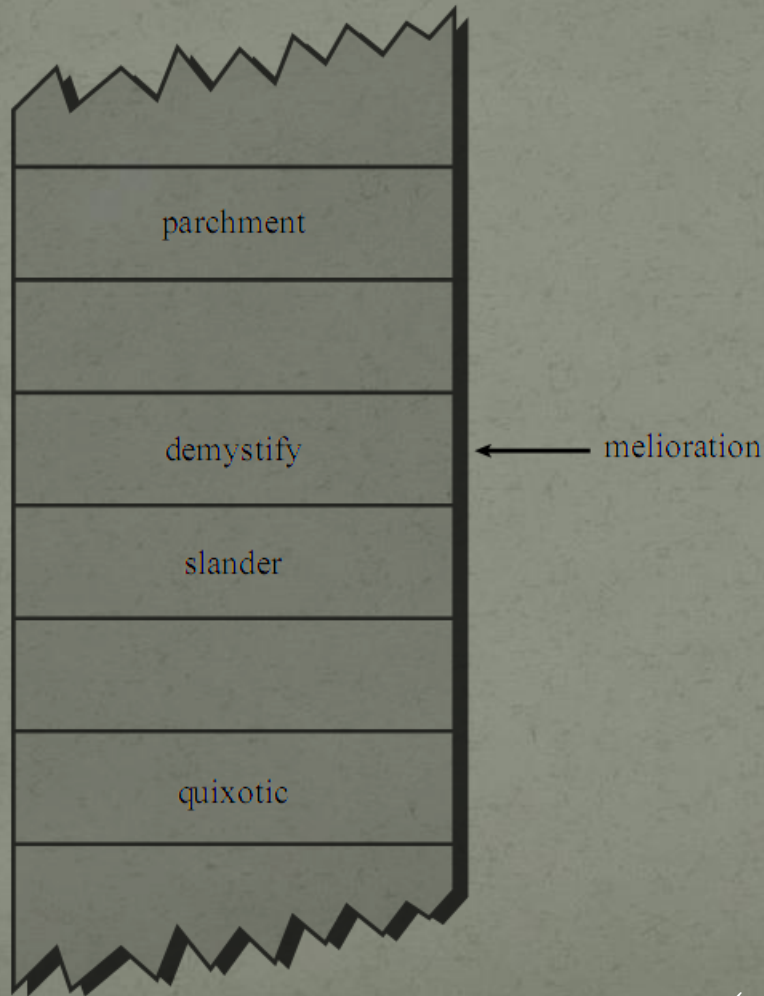
(hash function)



(example from R. Lafore book)

Hash Tables

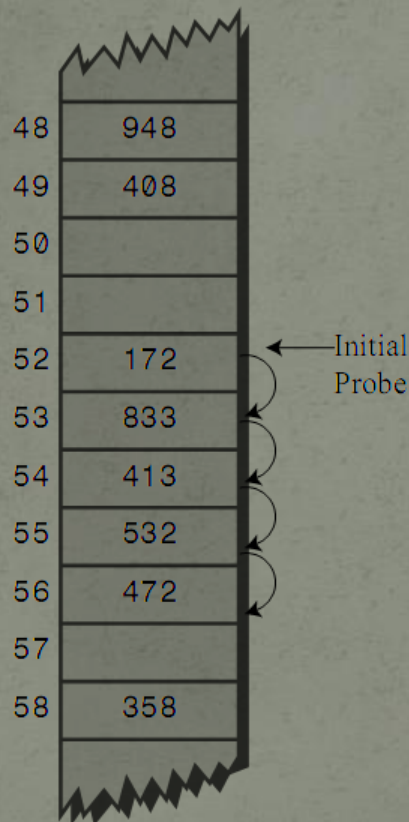
Collisions



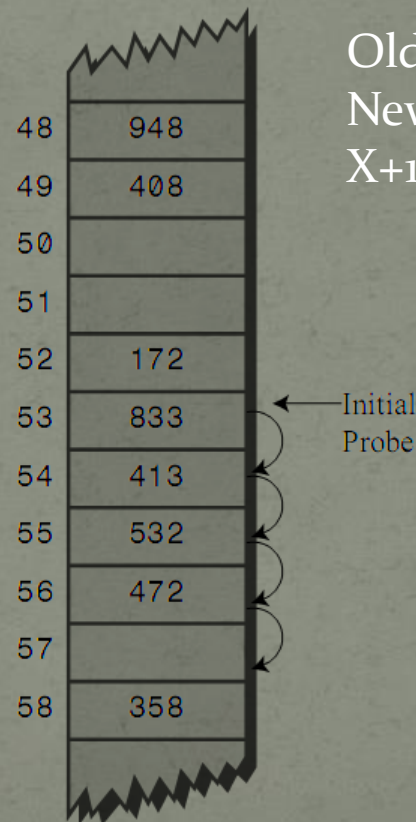
(example from R. Lafore book)

Hashing with Linear Probing

Collisions : inserting and finding



a) Successful search for 472



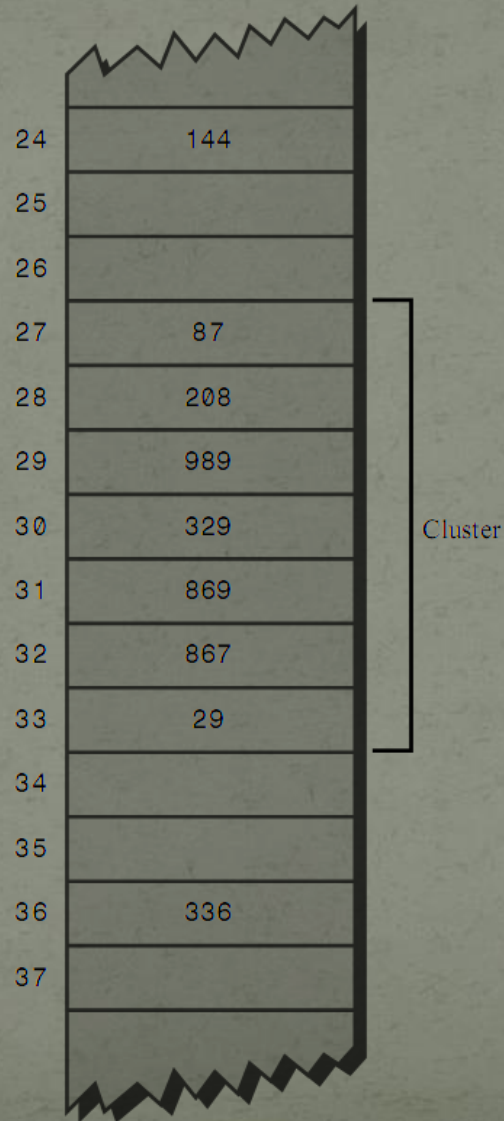
b) Unsuccessful search for 893

Old hash key = X
New hash key:
 $X+1, X+2, X+3, \dots, X+5$

(example from R. Lafore book)

Hashing with Linear Probing

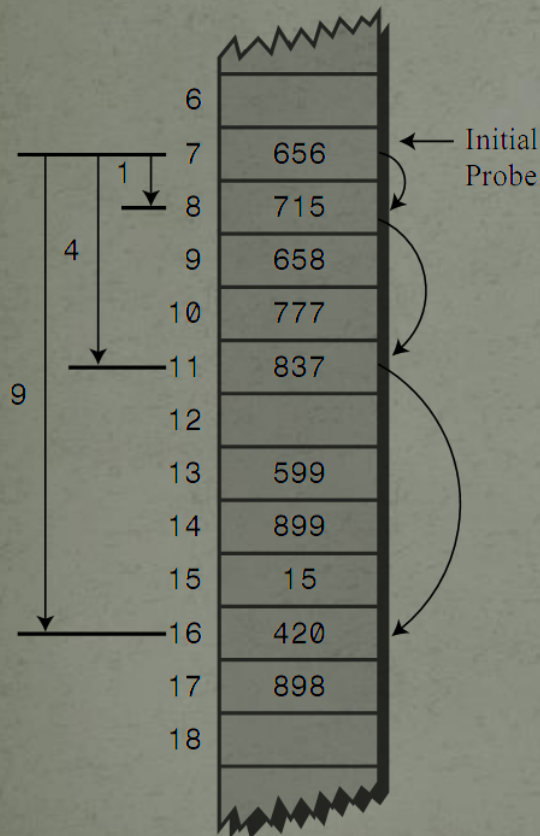
Clustering



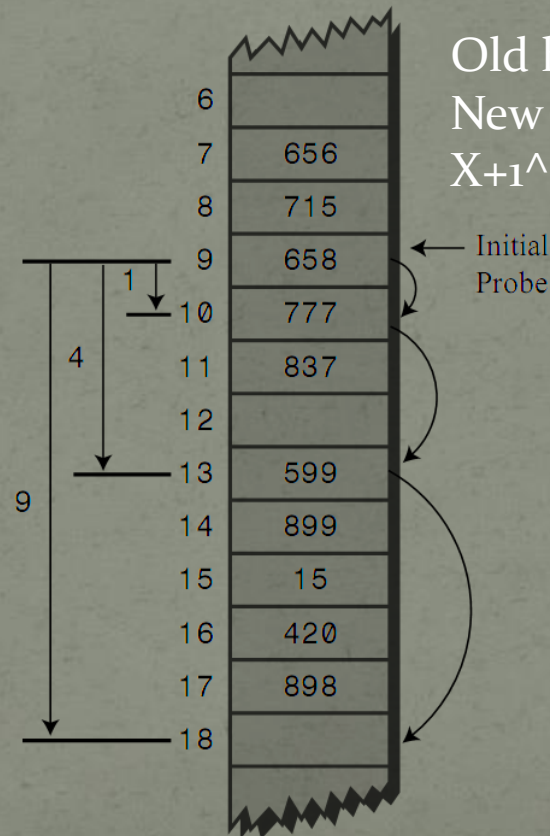
(example from R. Lafore book)

Hashing with Quadratic Probing

Quadrating Probing



a) Successful search for 420



b) Unsuccessful search for 481

Old hash key = X
New hash key:
 $X + 1^2, X + 2^2, X + 3^2, \dots, X + 5^2$

(example from R. Lafore book)

Hashing with Quadratic Probing

Quadrating Probing why to use?

- solve problem with **primary clustering** in linear probe

Can cause problem for keys which are hashing function return this same hash key

Example: Let's say for 184, 302, 420 and 544 hash function return 7. Then 302 will require a 1-step probe, 420 a 2-step-probe , 544 will require 3-step probe. This is a **secondary clustering**.

To sole problem with secondary clustering as well primary can be used: **double hashing**.

(example from R. Lafore book)

Hashing with Double Hashing

The basic idea is to use a **different** hash function a second time :

$\text{stepSize} = \text{constant} - (\text{key} \% \text{constant});$

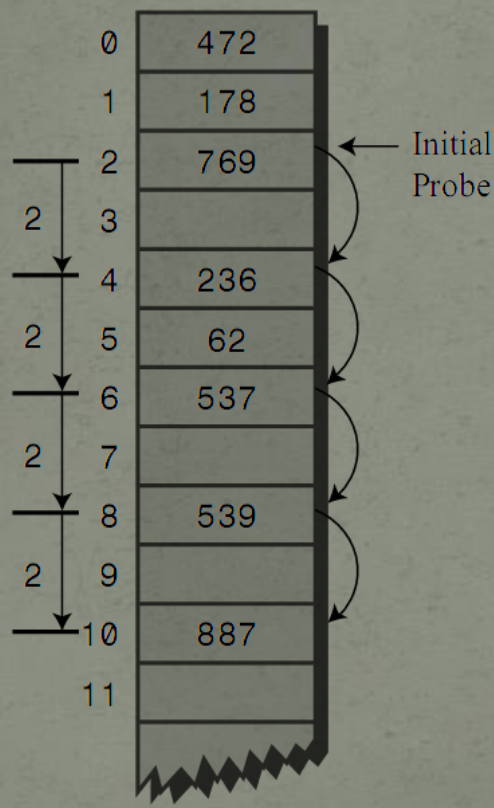
constant = prime number less than an array size; example:

$\text{stepSize} = 5 - (\text{key} \% 5);$

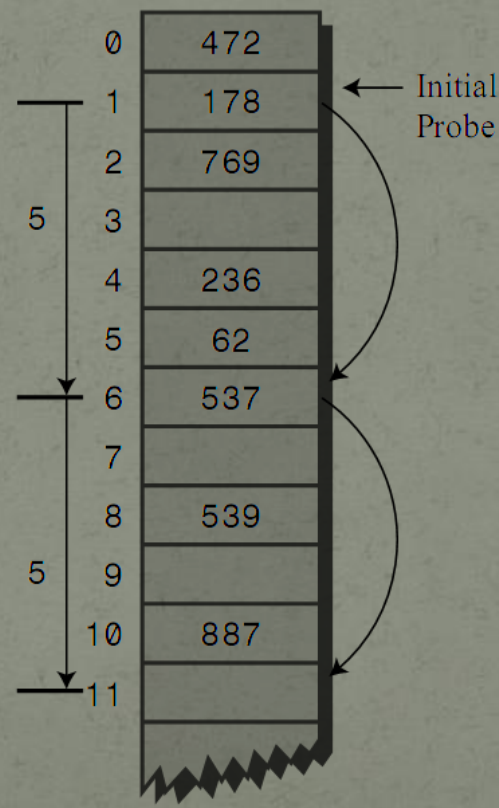
(example from R. Lafore book)

Hashing with Double Hashing

The basic idea is to use a **different** hash function a second time :



a) Successful
search for
887



b) Unsuccessful
search for 709
(example from R. Lafore book)

Hashing with Double Hashing

Double hashing requires an array size to be a prime number.

Example1 : arraySize = 15 (0, 1, 2,...,12, 13, 14), stepSize = 5

Initial index (hash key of particular key) = 0;

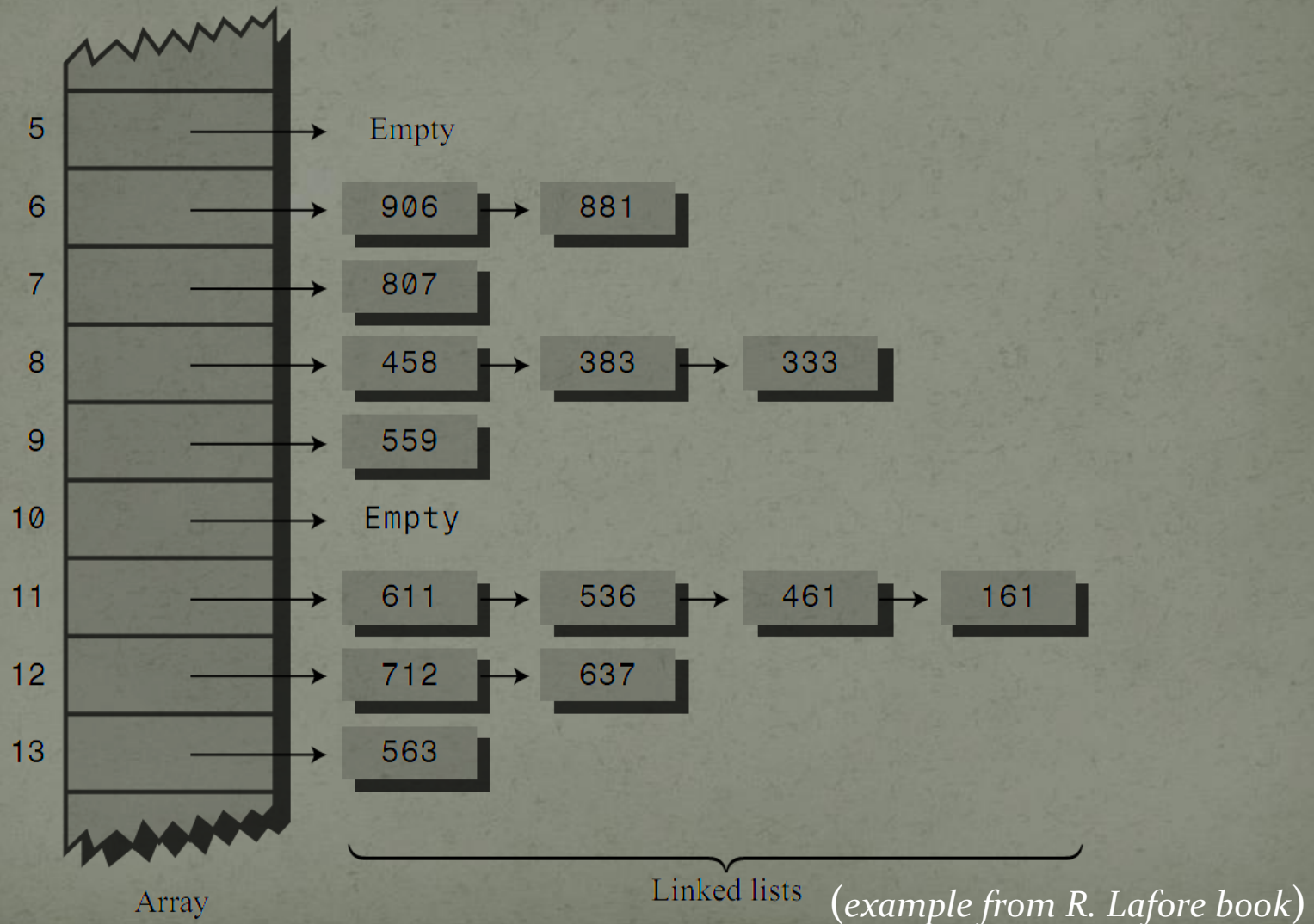
Probe sequence: 0, 5, 10, 0, 5, 10, ...

Example2 : arraySize = 13 (0, 1, 2,...,10, 11, 12), stepSize = 5

Initial index (hash key of particular key) = 0;

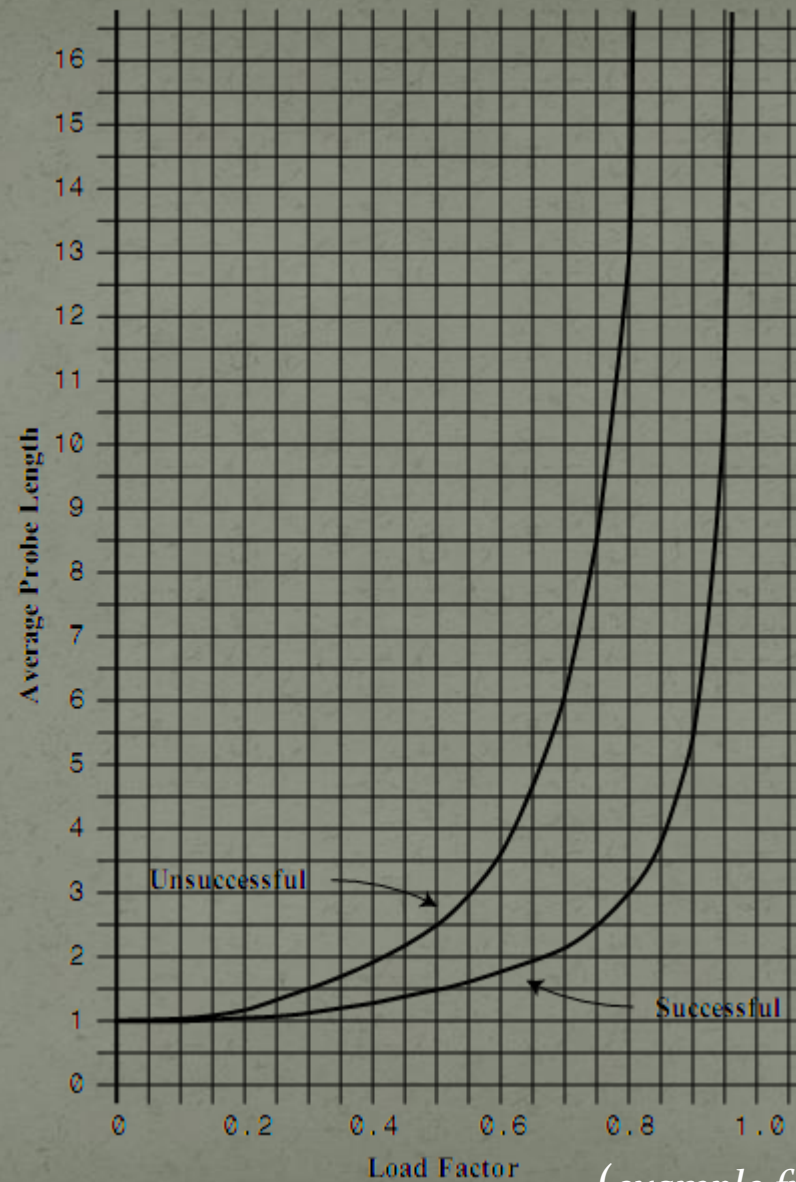
Probe sequence: 0, 5, 10, 2, 7, 12, 4, 9, 1, 6, 11, 3, ... (*all indexes of the array*).

Hashing with Separate Chaining



Efficiency

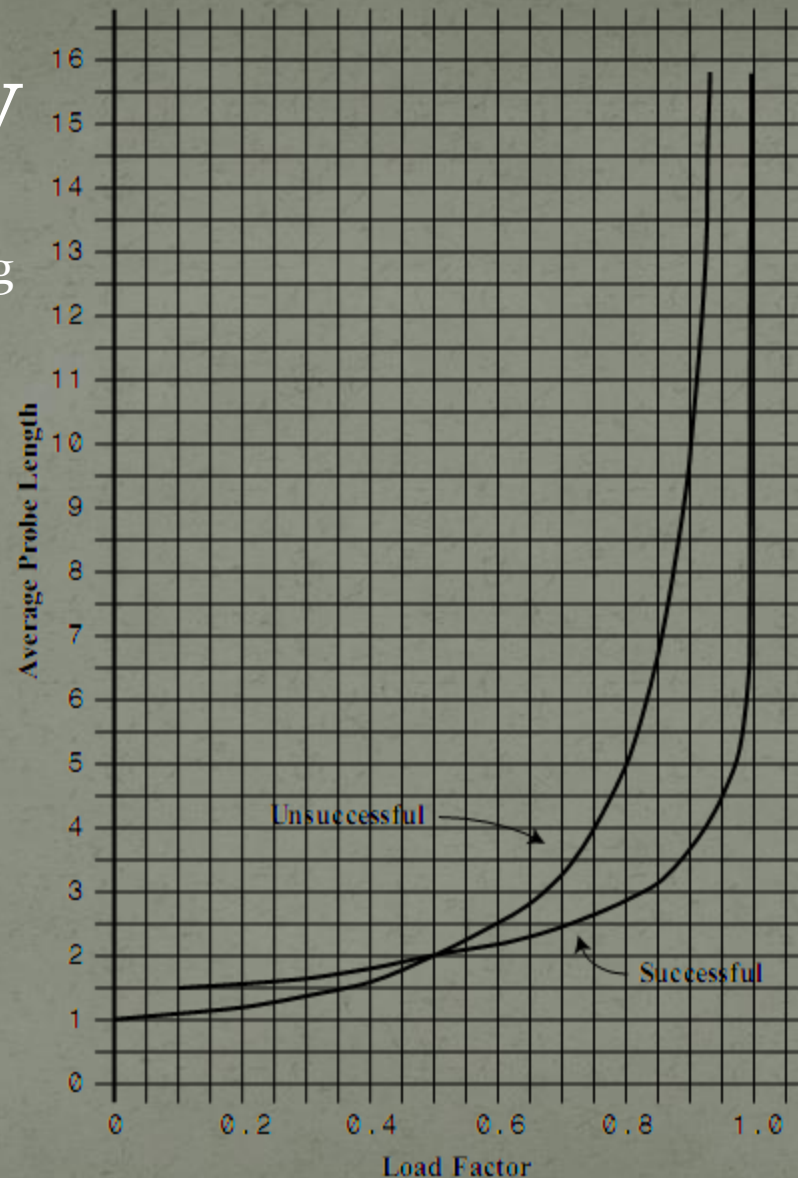
Linear Probing



(example from R. Lafore book)

Efficiency

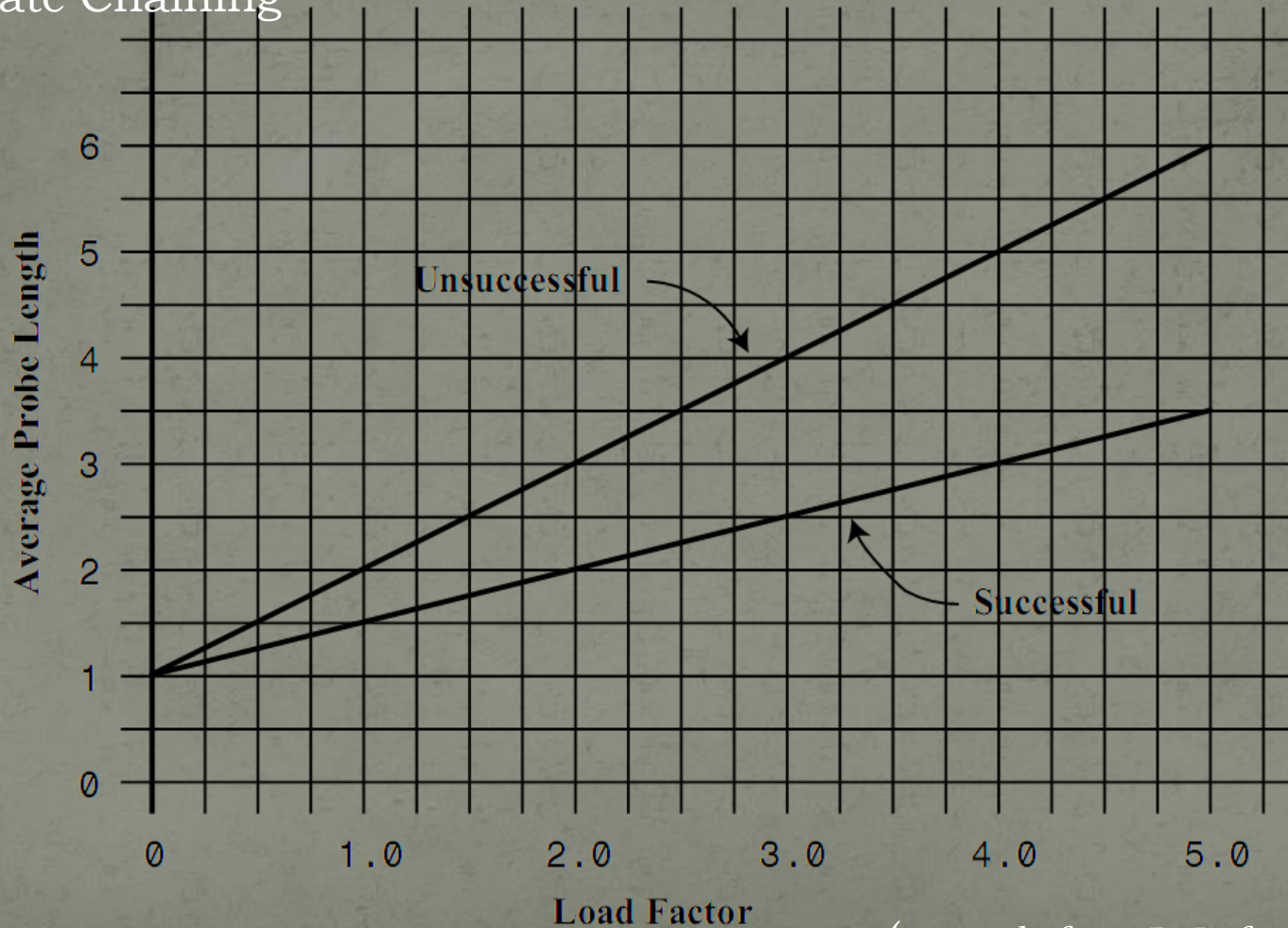
Quadratic Probing
and Double Hashing



(example from R. Lafore book)

Efficiency

Separate Chaining



(example from R. Lafore book)

