## KMO_1 03

### 1. Tworzenie list

```
In[1]:= {1, 2, 3}^3 - 1
Out[1]= {0, 7, 26}

In[2]:= b = {0, 3, 2.5}
Out[2]= {0, 3, 2.5}

In[3]:= b + 1
Out[3]= {1, 4, 3.5}

In[4]:= Table[n!, {n, 1, 7}]
Out[4]= {1, 2, 6, 24, 120, 720, 5040}

In[5]:= N[Log[%]]
Out[5]= {0., 0.693147, 1.79176, 3.17805, 4.78749, 6.57925, 8.52516}

In[6]:= Table[n^2, {n, 1, 7, 0.5}]
Out[6]= {1., 2.25, 4., 6.25, 9., 12.25,
  16., 20.25, 25., 30.25, 36., 42.25, 49.}

In[7]:= Array[a, 5]
Out[7]= {a[1], a[2], a[3], a[4], a[5]}

In[8]:= f[x_] := 3 x

In[9]:= Array[f, 5]
Out[9]= {3, 6, 9, 12, 15}

In[10]:= Range[3, 8, 0.25]
Out[10]= {3., 3.25, 3.5, 3.75, 4., 4.25, 4.5, 4.75, 5., 5.25, 5.5,
  5.75, 6., 6.25, 6.5, 6.75, 7., 7.25, 7.5, 7.75, 8.}
```

Ciag Fibonacciego:

```
In[11]:= a[1] = 1; a[2] = 1; a[n_] := a[n] = a[n - 1] + a[n - 2](* n≥ 3 *)

In[12]:= f = Table[a[i], {i, 1, 10}]
Out[12]= {1, 1, 2, 3, 5, 8, 13, 21, 34, 55}
```

## 2. Manipulowanie elementami list

In[13]:=
```
u = {1, 2, 7, 5, -4, 0};
```

In[14]:=
```
w = Part[u, {1, 3}]
```

Out[14]=
```
{1, 7}
```

In[15]:=
```
Last[w]
```

Out[15]=
```
7
```

In[16]:=
```
u[[{2, 5}]]
```

Out[16]=
```
{2, -4}
```

In[17]:=
```
lista = Table[Random[Integer, {0, 20}], {i, 1, 10}]
```

Out[17]=
```
{7, 19, 6, 2, 18, 15, 5, 3, 13, 2}
```

In[18]:=
```
Take[lista, {2, 7}]
```

Out[18]=
```
{19, 6, 2, 18, 15, 5}
```

In[19]:=
```
Rest[lista]
```

Out[19]=
```
{19, 6, 2, 18, 15, 5, 3, 13, 2}
```

In[20]:=
```
Most[lista]
```

Out[20]=
```
{7, 19, 6, 2, 18, 15, 5, 3, 13}
```

In[21]:=
```
Drop[lista, {5}]
```

Out[21]=
```
{7, 19, 6, 2, 15, 5, 3, 13, 2}
```

In[22]:=
```
Delete[lista, 4]
```

Out[22]=
```
{7, 19, 6, 18, 15, 5, 3, 13, 2}
```

In[23]:=
```
Delete[lista, {{4}, {3}}]
```

Out[23]=
```
{7, 19, 18, 15, 5, 3, 13, 2}
```

In[24]:=
```
lista
```

Out[24]=
```
{7, 19, 6, 2, 18, 15, 5, 3, 13, 2}
```

In[25]:=
```
lista = {1, a, {4, 3.5}, s};
```

Kasowanie liczb calkowitych z listy "lista" (wylacznie z "lista") :

In[26]:=
```
DeleteCases[lista, _Integer]
```

Out[26]=
```
{a, {4, 3.5}, s}
```

Z drugiego poziomu :

In[27]:=
```
DeleteCases[lista, _Integer, 2]
```
Out[27]=
```
{a, {3.5}, s}
```

Tylko z drugiego poziomu :

In[28]:=
```
DeleteCases[lista, _Integer, {2}]
```
Out[28]=
```
{1, a, {3.5}, s}
```

Usuwamy liste wewnetrzna :

In[29]:=
```
DeleteCases[lista, _List]
```
Out[29]=
```
{1, a, s}
```
In[30]:=
```
lista = Table[Prime[i], {i, 1, 12}]
```
Out[30]=
```
{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}
```
In[31]:=
```
Take[lista, {2, 8, 2}]
```
Out[31]=
```
{3, 7, 13, 19}
```
In[32]:=
```
Drop[lista, {2, 8, 2}]
```
Out[32]=
```
{2, 5, 11, 17, 23, 29, 31, 37}
```
In[33]:=
```
b1 = Table[N[Erf[i]], {i, 0, 2, 0.5}]
```
Out[33]=
```
{0., 0.5205, 0.842701, 0.966105, 0.995322}
```

gdzie Erf[] - funkcja bledu

In[34]:=
```
Prepend[b1, -1.3]
```
Out[34]=
```
{-1.3, 0., 0.5205, 0.842701, 0.966105, 0.995322}
```
In[35]:=
```
Append[b1, -3.4]
```
Out[35]=
```
{0., 0.5205, 0.842701, 0.966105, 0.995322, -3.4}
```
In[36]:=
```
b1
```
Out[36]=
```
{0., 0.5205, 0.842701, 0.966105, 0.995322}
```
In[37]:=
```
AppendTo[b1, -3.4]
```
Out[37]=
```
{0., 0.5205, 0.842701, 0.966105, 0.995322, -3.4}
```
In[38]:=
```
b1
```
Out[38]=
```
{0., 0.5205, 0.842701, 0.966105, 0.995322, -3.4}
```
In[39]:=
```
Insert[b1, a, 4]
```
Out[39]=
```
{0., 0.5205, 0.842701, a, 0.966105, 0.995322, -3.4}
```
In[40]:=
```
Insert[b1, a, {{2}, {4}, {6}}]
```
Out[40]=
```
{0., a, 0.5205, 0.842701, a, 0.966105, 0.995322, a, -3.4}
```

In[41]:=
```
b1[[5]] = -10;
```

In[42]:=
```
b1
```

Out[42]=
```
{0., 0.5205, 0.842701, 0.966105, -10, -3.4}
```

In[43]:=
```
lista = {1, 2, 3, 4};
```

In[44]:=
```
PadLeft[lista, 8]
```

Out[44]=
```
{0, 0, 0, 0, 1, 2, 3, 4}
```

In[45]:=
```
PadLeft[lista, 8, -1]
```

Out[45]=
```
{-1, -1, -1, -1, 1, 2, 3, 4}
```

In[46]:=
```
PadLeft[lista, 8, {-1, -2}]
```

Out[46]=
```
{-1, -2, -1, -2, 1, 2, 3, 4}
```

In[47]:=
```
PadLeft[lista, 9, lista]
```

Out[47]=
```
{4, 1, 2, 3, 4, 1, 2, 3, 4}
```

In[48]:=
```
PadLeft[lista, 12, {a, b}, 3]
```

Out[48]=
```
{{0, 3, 2.5}, a, {0, 3, 2.5}, a,
  {0, 3, 2.5}, 1, 2, 3, 4, a, {0, 3, 2.5}, a}
```

In[49]:=
```
PadLeft[lista, 12, {a, b}]
```

Out[49]=
```
{a, {0, 3, 2.5}, a, {0, 3, 2.5}, a,
  {0, 3, 2.5}, a, {0, 3, 2.5}, 1, 2, 3, 4}
```

In[50]:=
```
Join[{1, a, 3}, {e, 3, c}, {1, 2, 3, 4}]
```

Out[50]=
```
{1, a, 3, e, 3, c, 1, 2, 3, 4}
```

In[51]:=
```
Union[{1, a, 3}, {e, 3, c}, {1, 2, 3, 4}]
```

Out[51]=
```
{1, 2, 3, 4, a, c, e}
```

## 3. Instrukcje opisujace listy

Wielomiany Czebeszewa :

In[52]:=
```
c = Table[ChebyshevT[i, x], {i, 1, 5}]
```

Out[52]=
$$\{x, -1 + 2 x^2, -3 x + 4 x^3, 1 - 8 x^2 + 8 x^4, 5 x - 20 x^3 + 16 x^5\}$$

In[53]:=
```
hc = Table[c[[i]] - HermiteH[i, x], {i, 1, Length[c]}]
```

Out[53]=
$$\{-x, 1 - 2 x^2, 9 x - 4 x^3, -11 + 40 x^2 - 8 x^4, -115 x + 140 x^3 - 16 x^5\}$$

In[54]:=
```
MemberQ[hc, x^2 + 1]
```

Out[54]=
```
False
```

Czy ’x^2 + 1’ nie wystepuje w hc :

In[55]:=
```
FreeQ[hc, x^2 + 1]
```

Out[55]=
```
True
```

In[56]:=
```
TableForm[hc]
```

Out[56]//TableForm=

$-x$

$1 - 2\,x^2$

$9\,x - 4\,x^3$

$-11 + 40\,x^2 - 8\,x^4$

$-115\,x + 140\,x^3 - 16\,x^5$

In[57]:=
```
los = Table[Random[Integer, {-5, 5}], {i, 1, 10}]
```

Out[57]=
```
{0, -4, -1, -4, 1, 1, 3, -1, 4, -3}
```

Ile ’0’ wystepuje w los :

In[58]:=
```
Count[los, 0]
```

Out[58]=
```
1
```

In[59]:=
```
Count[los, -2]
```

Out[59]=
```
0
```

In[60]:=
```
Position[los, 3]
```

Out[60]=
```
{{7}}
```

In[61]:=
```
w = {{1, 2}, {3, 4}};
```

In[62]:=
```
ColumnForm[w]
```

Out[62]=
```
{1, 2}
{3, 4}
```

In[63]:=
```
TableForm[w]
```

Out[63]//TableForm=

```
1 2
3 4
```

## 4. Operacje matematyczne na listach

In[64]:=
```
licz = {1, 6, 8, -3, 5, -1};
```

In[65]:=
```
Apply[Plus, licz]
```

Out[65]=
```
16
```

In[66]:=
```
Apply[Times, licz]
```
Out[66]= 720

In[67]:=
```
Clear[f]
```

In[68]:=
```
f[x_] := x^3 - 1
```

In[69]:=
```
f[licz]
```
Out[69]= {0, 215, 511, -28, 124, -2}

In[70]:=
```
MapAt[f, licz, 4](*tylko na 4-ty rlement *)
```
Out[70]= {1, 6, 8, -28, 5, -1}

In[71]:=
```
MapAt[f, licz, {{2}, {3}, {6}}]
```
Out[71]= {1, 215, 511, -3, 5, -2}

In[72]:=
```
a = Table[i + j, {i, 1, 3}, {j, 1, 4}]
```
Out[72]= {{2, 3, 4, 5}, {3, 4, 5, 6}, {4, 5, 6, 7}}

Suma elementow w poszczegolnych kolumnach :

In[73]:=
```
Total[a]
```
Out[73]= {9, 12, 15, 18}

Suma wszystkich elementow :

In[74]:=
```
Total[a, 2]
```
Out[74]= 54

In[75]:=
```
Total[a, Depth[a]]
```
Out[75]= 54

## 5. Listy zlozone

In[76]:=
```
pr = Table[i, {i, 1, 6}]
```
Out[76]= {1, 2, 3, 4, 5, 6}

In[77]:=
```
Partition[pr, 2]
```
Out[77]= {{1, 2}, {3, 4}, {5, 6}}

In[78]:=
```
Partition[pr, 3]
```
Out[78]= {{1, 2, 3}, {4, 5, 6}}

Z przesunieciem o jeden element :

In[79]:=
```
lz = Partition[pr, 3, 1]
```
Out[79]= {{1, 2, 3}, {2, 3, 4}, {3, 4, 5}, {4, 5, 6}}

In[80]:=
```
Clear[a, b, c]; test = {a, a, a, b, b, c, c, a, a, b, c, c, c};
```

In[81]:=
```
Split[test]
```

Out[81]=
```
{{a, a, a}, {b, b}, {c, c}, {a, a}, {b}, {c, c, c}}
```

In[82]:=
```
n1 = {{2, 3, 5, 7}, {11, 13, 17, 19}, {23, 29, 31, 37}};
```

wybiera z poziomow 2 i 3 elementy od 1 do 3 :

In[83]:=
```
Take[n1, {2, 3}, {1, 3}]
```

Out[83]=
```
{{11, 13, 17}, {23, 29, 31}}
```

In[84]:=
```
Drop[n1, {1, 2}, {3, 4}]
```

Out[84]=
```
{{23, 29}}
```

In[85]:=
```
a = {{1, 2}, {3, 4}};
```

In[86]:=
```
PadRight[a, {4, 4}]
```

Out[86]=
```
{{1, 2, 0, 0}, {3, 4, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```

In[87]:=
```
MatrixForm[%]
```

Out[87]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[88]:=
```
PadRight[a, {4, 3}]
```

Out[88]=
```
{{1, 2, 0}, {3, 4, 0}, {0, 0, 0}, {0, 0, 0}}
```

In[89]:=
```
PadRight[a, {4, 4}, {0, 0}, {1, 1}]
```

Out[89]=
```
{{0, 0, 0, 0}, {0, 1, 2, 0}, {0, 3, 4, 0}, {0, 0, 0, 0}}
```

In[90]:=
```
PadRight[a, {4, 4}, {0, 0}, {1, 0}]
```

Out[90]=
```
{{0, 0, 0, 0}, {1, 2, 0, 0}, {3, 4, 0, 0}, {0, 0, 0, 0}}
```

In[91]:=
```
PadRight[a, {4, 4}, {3, 0}, {1, 0}]
```

Out[91]=
```
{{3, 0, 3, 0}, {1, 2, 3, 0}, {3, 4, 3, 0}, {3, 0, 3, 0}}
```

In[92]:=
```
PadRight[a, {4, 4}, {3, 0}, {1, 1}]
```

Out[92]=
```
{{0, 3, 0, 3}, {0, 1, 2, 3}, {0, 3, 4, 3}, {0, 3, 0, 3}}
```

In[93]:=
```
PadRight[a, {5, 4}, {3, 2}, {1, 0}]
```

Out[93]=
```
{{3, 2, 3, 2}, {1, 2, 3, 2},
  {3, 4, 3, 2}, {3, 2, 3, 2}, {3, 2, 3, 2}}
```

In[94]:=
```
lz
```
Out[94]=
```
{{1, 2, 3}, {2, 3, 4}, {3, 4, 5}, {4, 5, 6}}
```
In[95]:=
```
Flatten[lz]
```
Out[95]=
```
{1, 2, 3, 2, 3, 4, 3, 4, 5, 4, 5, 6}
```
In[96]:=
```
g = {{1, 2}, {a1, {1}, {1, {0}, 6}}};
```
In[97]:=
```
Flatten[g, 2]
```
Out[97]=
```
{1, 2, a1, 1, 1, {0}, 6}
```
In[98]:=
```
Flatten[g, 3]
```
Out[98]=
```
{1, 2, a1, 1, 1, 0, 6}
```
In[99]:=
```
FlattenAt[g, 2]
```
Out[99]=
```
{{1, 2}, a1, {1}, {1, {0}, 6}}
```
In[100]:=
```
FlattenAt[g, 1]
```
Out[100]=
```
{1, 2, {a1, {1}, {1, {0}, 6}}}
```
In[101]:=
```
FlattenAt[g, {{1}, {2}}]
```
Out[101]=
```
{1, 2, a1, {1}, {1, {0}, 6}}
```

## 6. Wektory

In[102]:=
```
u = {x, y, z}
```
Out[102]=
```
{x, y, z}
```
In[103]:=
```
u[[2]]
```
Out[103]=
```
y
```
In[104]:=
```
p * u + q
```
Out[104]=
```
{q + p x, q + p y, q + p z}
```
In[105]:=
```
{x, y}.{p, q}
```
Out[105]=
```
p x + q y
```

Iloczyn wektorowy :

In[106]:=
```
Clear[a, b, c, x, y, z]
```
In[107]:=
```
Cross[{a, b, c}, {x, y, z}]
```
Out[107]=
```
{-c y + b z, c x - a z, -b x + a y}
```
In[108]:=
```
VectorQ[{1, 23, 7}]
```
Out[108]=
```
True
```

In[109]:= **VectorQ[{1, {3}, 7}]**

Out[109]= False

In[110]:= **v = {1, 5, -1, 3};**

In[111]:= **Norm[v]**

Out[111]= 6

p - norma :

$$||v||_p = \left(\sum_{i=i}^{n} (v_i)^p\right)^{1/p}$$

In[112]:= **Norm[v, 2]**

Out[112]= 6

In[113]:= **Norm[v, 3]**

Out[113]= $154^{1/3}$

In[114]:= **Norm[v, ∞]**

Out[114]= 5

## ■ 7. Macierze

In[115]:= **m = {{a, b}, {c, d}}**

Out[115]= {{a, b}, {c, d}}

In[116]:= **MatrixForm[m]**

Out[116]//MatrixForm=

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

In[117]:= **m[[1, 2]]**

Out[117]= b

In[118]:= **n = {{2, 3, 4}, {0, 1, 2}}**

Out[118]= {{2, 3, 4}, {0, 1, 2}}

In[119]:= **m.n**

Out[119]= {{2 a, 3 a + b, 4 a + 2 b}, {2 c, 3 c + d, 4 c + 2 d}}

In[120]:= **v = {x, y};**

Mathematica sama interpretuje czy v jest wektorem wierszowym czy kolumnowym :

In[121]:=
```
m.v
```
Out[121]=
$$\{a\,x + b\,y,\ c\,x + d\,y\}$$

In[122]:=
```
v.m
```
Out[122]=
$$\{a\,x + c\,y,\ b\,x + d\,y\}$$

## 8. Dzialania na macierzach

In[123]:=
```
a = {{2, 0, 1, 4}, {-1, 7, 3, 2}, {0, 4, 6, -3}}
```
Out[123]=
$$\{\{2, 0, 1, 4\}, \{-1, 7, 3, 2\}, \{0, 4, 6, -3\}\}$$

In[124]:=
```
MatrixForm[a]
```
Out[124]//MatrixForm=
$$\begin{pmatrix} 2 & 0 & 1 & 4 \\ -1 & 7 & 3 & 2 \\ 0 & 4 & 6 & -3 \end{pmatrix}$$

In[125]:=
```
a[[2, 4]]
```
Out[125]=
$$2$$

wypisanie wiersza :

In[126]:=
```
a[[3]]
```
Out[126]=
$$\{0, 4, 6, -3\}$$

In[127]:=
```
a[[3, All]]
```
Out[127]=
$$\{0, 4, 6, -3\}$$

Pojedyncza kolumna :

In[128]:=
```
a[[All, 2]]
```
Out[128]=
$$\{0, 7, 4\}$$

In[129]:=
```
m = Table[1/(i + j + 1), {i, 1, 3}, {j, 1, 3}]
```
Out[129]=
$$\left\{\left\{\frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}, \left\{\frac{1}{4}, \frac{1}{5}, \frac{1}{6}\right\}, \left\{\frac{1}{5}, \frac{1}{6}, \frac{1}{7}\right\}\right\}$$

In[130]:=
```
MatrixForm[m]
```
Out[130]//MatrixForm=
$$\begin{pmatrix} \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{pmatrix}$$

In[131]:=
```
n = m + IdentityMatrix[3]
```
Out[131]=
$$\left\{\left\{\frac{4}{3}, \frac{1}{4}, \frac{1}{5}\right\}, \left\{\frac{1}{4}, \frac{6}{5}, \frac{1}{6}\right\}, \left\{\frac{1}{5}, \frac{1}{6}, \frac{8}{7}\right\}\right\}$$

In[132]:=
```
n - m
```
Out[132]=
```
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

In[133]:=
```
30 * m
```
Out[133]=
$$\left\{\left\{10, \frac{15}{2}, 6\right\}, \left\{\frac{15}{2}, 6, 5\right\}, \left\{6, 5, \frac{30}{7}\right\}\right\}$$

In[134]:=
```
ArrayQ[n]
```
Out[134]=
```
True
```

In[135]:=
```
ArrayDepth[a]
```
Out[135]=
```
2
```

Macierz odwrotna :

In[136]:=
```
m1 = Inverse[m]
```
Out[136]=
```
{{300, -900, 630}, {-900, 2880, -2100}, {630, -2100, 1575}}
```

In[137]:=
```
m.m1
```
Out[137]=
```
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

In[138]:=
```
a = {{1, 2, 3}, {-1, 3, 1}, {0, 1, 0}};
```

In[139]:=
```
MatrixForm[a]
```
Out[139]//MatrixForm=
$$\begin{pmatrix} 1 & 2 & 3 \\ -1 & 3 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

In[140]:=
```
at = Transpose[a]; MatrixForm[at]
```
Out[140]//MatrixForm=
$$\begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 1 \\ 3 & 1 & 0 \end{pmatrix}$$

In[141]:=
```
{Det[a], Det[at]}
```
Out[141]=
```
{-4, -4}
```

Slad :

In[142]:=
```
Tr[a]
```
Out[142]=
```
4
```

Glowna przekatna :

In[143]:=
```
Tr[a, List]
```

Out[143]=
```
{1, 3, 0}
```

Lista wszystkich minorow stopnia drugiego :

In[144]:=
```
Minors[a, 2]
```

Out[144]=
```
{{5, 4, -7}, {1, 0, -3}, {-1, 0, -1}}
```

In[145]:=
```
MatrixForm[a]
```

Out[145]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 3 \\ -1 & 3 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

In[146]:=
```
MatrixForm[Inverse[Inverse[a], Modulus -> 3]](*modulo 3*)
```

Out[146]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

In[147]:=
```
Table[{i, Mod[i, 3]}, {i, -10, 10}]
```

Out[147]=
```
{{-10, 2}, {-9, 0}, {-8, 1}, {-7, 2},
  {-6, 0}, {-5, 1}, {-4, 2}, {-3, 0}, {-2, 1},
  {-1, 2}, {0, 0}, {1, 1}, {2, 2}, {3, 0}, {4, 1},
  {5, 2}, {6, 0}, {7, 1}, {8, 2}, {9, 0}, {10, 1}}
```

In[148]:=
```
Table[{i, Mod[i, 5]}, {i, -10, 10}]
```

Out[148]=
```
{{-10, 0}, {-9, 1}, {-8, 2}, {-7, 3},
  {-6, 4}, {-5, 0}, {-4, 1}, {-3, 2}, {-2, 3},
  {-1, 4}, {0, 0}, {1, 1}, {2, 2}, {3, 3}, {4, 4},
  {5, 0}, {6, 1}, {7, 2}, {8, 3}, {9, 4}, {10, 0}}
```

## ■ 9. Wartosci i wektory wlasne

Wielomian charakterystyczny macierzy a :

In[149]:=
```
a = {{1, 0, -1}, {0, 1, 2}, {1, 1, -1}};
```

In[150]:=
```
wiel = CharacteristicPolynomial[a, x]
```

Out[150]= $-2 + 2x + x^2 - x^3$

In[151]:=
```
-2 + 2 x + x^2 - x^3
```

Out[151]= $-2 + 2x + x^2 - x^3$

Pierwiastki wielomianu == wartosci wlasne :

In[152]:=
```
Solve[wiel == 0, x]
```

Out[152]=
```
{{x → 1}, {x → -√2}, {x → √2}}
```

In[153]:=
```
{{x → 1}, {x → -√2}, {x → √2}}
```

Out[153]=
```
{{x → 1}, {x → -√2}, {x → √2}}
```

In[154]:=
```
Eigenvalues[a]
```

Out[154]=
```
{-√2, √2, 1}
```

In[155]:=
```
{-√2, √2, 1}
```

Out[155]=
```
{-√2, √2, 1}
```

Wektory wlasne :

In[156]:=
```
Eigenvectors[a]
```

Out[156]=
```
{{-1 + √2, 2 - 2 √2, 1}, {-1 - √2, 2 + 2 √2, 1}, {-1, 1, 0}}
```

In[157]:=
```
{{-1 + √2, 2 - 2 √2, 1}, {-1 - √2, 2 + 2 √2, 1}, {-1, 1, 0}}
```

Out[157]=
```
{{-1 + √2, 2 - 2 √2, 1}, {-1 - √2, 2 + 2 √2, 1}, {-1, 1, 0}}
```

In[158]:=
```
Eigensystem[a] // N(* {wartosciwlasne,wektorywlasne} *)
```

Out[158]=
```
{{-1.41421, 1.41421, 1.},
 {{0.414214, -0.828427, 1.}, {-2.41421, 4.82843, 1.}, {-1., 1., 0.}}}
```

In[159]:=
```
{{-1.41421, 1.41421, 1.}, {{0.414214, -0.828427, 1.},
  {-2.41421, 4.82843, 1.}, {-1., 1., 0.}}}
```

Out[159]=
```
{{-1.41421, 1.41421, 1.},
 {{0.414214, -0.828427, 1.}, {-2.41421, 4.82843, 1.}, {-1., 1., 0.}}}
```

## 10. Uklady rownan liniowych

In[160]:=
```
a = {{1, 2, 0}, {-1, 1, 1}, {0, 1, 2}};
```

In[161]:=
```
b = {3, 1, 3};
```

Rozwiazywanie ukldau A x = b :

In[162]:=
```
LinearSolve[a, b]
```

Out[162]=
```
{1, 1, 1}
```

In[163]:=
```
{1, 1, 1}
```

Out[163]=
```
{1, 1, 1}
```

In[164]:=
```
m = {{2, -2, 0}, {-1, 1, 1}, {-1, 1, 0}};
```

Czy istnieje rozwiazanie niezerowe (Det[m] == 0) :

In[165]:=
```
Det[m]
```

Out[165]=
```
0
```

In[166]:=
```
0
```

Out[166]=
```
0
```

Wektory bazowe przestrzeni rozwiazan :

In[167]:=
```
n = NullSpace[m]
```

Out[167]=
```
{{1, 1, 0}}
```

In[168]:=
```
{{1, 1, 0}}
```

Out[168]=
```
{{1, 1, 0}}
```

In[169]:=
```
m.{1, 1, 0}
```

Out[169]=
```
{0, 0, 0}
```

In[170]:=
```
{0, 0, 0}
```

Out[170]=
```
{0, 0, 0}
```

Z dowolnym jednym parametrem :

In[171]:=
```
roz = t*n[[1]]
```

Out[171]=
```
{t, t, 0}
```

In[172]:=
```
{t, t, 0}
```

Out[172]=
```
{t, t, 0}
```

In[173]:=
```
m.roz
```

Out[173]=
```
{0, 0, 0}
```

In[174]:=
```
{0, 0, 0}
```

Out[174]=
```
{0, 0, 0}
```

Redukcja wierszowa :

In[175]:=
```
mr = RowReduce[m]
```

Out[175]=
```
{{1, -1, 0}, {0, 0, 1}, {0, 0, 0}}
```

In[176]:=
```
{{1, -1, 0}, {0, 0, 1}, {0, 0, 0}}
```

Out[176]=
```
{{1, -1, 0}, {0, 0, 1}, {0, 0, 0}}
```

In[177]:=
```
NullSpace[mr]
```

Out[177]=
```
{{1, 1, 0}}
```

In[178]:=
```
{{1, 1, 0}}
```

Out[178]=
```
{{1, 1, 0}}
```

*Opracowano na podstawie ksiazki Grzegorz Drwal, (+inni) - "Mahematica 5"*

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